

VIBRATION OF AN ELL-SHAPED UNIFORM  
FILAMENT WITH CLAMPED ENDS

By

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# United States Naval Postgraduate School



## THESIS

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December 1970

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T137105



Vibration of an Ell-shaped Uniform  
Filament with Clamped Ends

by

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Submitted in partial fulfillment of the  
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the  
NAVAL POSTGRADUATE SCHOOL  
December 1970

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## ABSTRACT

An analysis is made of in-plane and out-of-plane vibrations of a uniform filament which is bent at right angles and the ends of which are fixed. Shear deflection and rotatory inertia are neglected. Curve sheets are presented from which one may find the first several frequencies of vibration in terms of the geometrical and physical variables. An important application is to the vibrations of piping systems.

Also, as a separate project, a vibration program written by G. Fink for the CDC 1604 computer and modified by Y. S. Kim, has been rewritten for use with the IBM 360/67 computer.





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## ACKNOWLEDGEMENT

The author wishes to express his appreciation to Dr. J. E. Brock for the invaluable aid and counsel which he has offered during the preparation of this thesis.



## I. INTRODUCTION

### A. GENERAL

In order to assure long time integrity of nuclear piping, the applicable codes, ANSI B31.7 - 1969 Code for Nuclear Power Piping and forthcoming edition of Section III of the ASME Boiler and Pressure Vessel Code, call for seismic analysis of such piping and require that the severity of the effects of seismically induced vibration be maintained below specified acceptable levels. Generally the problem of vibration analysis of the major piping systems is so complex that a major computational effort is required and this demands digital computer analysis using quite sophisticated programs. However, in the early stages of a design, when alternate layouts are being investigated, it is useful to be able to make some reasonable estimates of what a more elaborate analysis would be likely to reveal.

A recent document [3] concerned with assisting a designer to understand and comply with ANSI B31.7 contains material summarizing the vibration characteristics of several simple piping configurations. It suggests how the designer might employ this sort of information to assist him in estimating the vibration characteristics of more complicated configurations. This material should prove to be quite useful.

However, the scope of the configurations about which such information is available is quite limited. One of the





basic configurations with which a designer deals is a simple ell-shaped configuration. It was decided that it would be quite useful to determine and present in a concise form the significant results relating to the vibration of such a configuration. Specifically, it is assumed that the two ends of the configuration are perfectly fixed or clamped, that the pipe is of uniform properties along its entire length and is of rather small diameter compared to its length (hence the word "filament" in the title of this thesis) so that shear deflection and rotatory inertia could be neglected, and that the right angle junction of the two legs of the ell is small compared to the overall dimensions of the ell itself, so that it is not necessary to consider the properties of the bend or elbow that would exist in practice at the junction.

Such results are obtained and given herein. The basic parameter is the ratio of the length of the shorter leg to that of the longer leg; this parameter is designated by the Greek letter rho in what follows. A second important parameter, Greek beta in what follows, is required to relate the coupling that takes place between bending and torsion in the case of what are called out-of-plane vibrations.

This simple ell-shaped configuration has the great virtue of being such that the vibration problem is naturally divided into two disjoint sub-parts. One such part involves only displacements which lie in the plane of the ell; no torsion is involved in these cases. The other part involves



both bending and torsion, but the motion of any point on the axis of the filament is only perpendicular to the plane of the ell. This gives rise to the two terms used in what follows: "in-plane vibrations" and "out-of-plane vibrations."

## B. ASSUMPTIONS AND LIMITATIONS

It is well to repeat here specifically the assumptions and limitations involved in this investigation.

1. The configuration is formed from originally straight uniform elastic members which meet at a right angle.
2. The angle at which the two members meet remains a right angle throughout the deformation of the configuration.
3. The ends of the configuration are assumed to be perfectly clamped.
4. The effect of shear deformation has been neglected. Only bending and torsional compliances are considered and deformations are assumed to be "small."
5. Rotatory inertia has been neglected.
6. Poisson's ratio has been taken to be 0.3.

Additionally, there are limitations implicit in the fact that values of the roots of the equations which are later solved are obtained only to an accuracy of one part in one million parts and in the fact that the computational programs used a mantissa having 56 bits. These limitations resulted in being able to obtain only about the lowest



thirteen frequencies for a particular configuration. Also, only a few values of  $\beta$  were investigated and there are some difficulties in the use of linear interpolation for other values of  $\beta$ .

### C. NOTATION

A, B, C, D, F, H Constants in equation (2-5) and (2-14)

E Modulus of elasticity

G Shear modulus of elasticity

$g_1, g_2, g_3, g_4$  Functions defined in equation (2-6)

I Moment of inertia of cross section

J Polar moment of inertia

L Length of pipe

M Bending moment

P Defined in equation (2-4)

t Time

T Torque

V Shear force

x Distance from fixed end

y Lateral deflection

Y Lateral deflection mode shape

$Y'$  Slope

$\alpha$  Defined in equation (2-13)

$\beta$  Defined in equation (2-29)

$\gamma$  Phase angle in equation (2-11)

$\theta$  Angle of twist

$\phi$  Angle of twist mode shape



$\psi$	Constant in equation (2-2)
$\mu$	Mass per unit length
$\nu$	Mass moment of inertia per unit length
$\rho$	Length ratio
$\omega$	Circular frequency





## II. THEORETICAL DEVELOPMENTS

### A. BENDING ANALYSIS

Neglecting the effects of shear deflection and rotatory inertia the equation governing lateral vibration of a uniform beam is

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = 0 \quad (2-1)$$

where  $\mu$  is mass per unit length,  $EI$  is flexural rigidity,  $y$  is lateral deflection and  $t$  is time. Assuming harmonic oscillation with circular frequency  $\omega$ , the solution of equation (2-1) can be written in the following form:

$$y(x,t) = Y(x) \cos(\omega t + \psi) \quad (2-2)$$

where  $\psi$  is a constant and  $Y$  is a function of  $x$  alone. Substitution of equation (2-2) into equation (2-1) yields

$$\frac{d^4 Y(x)}{dx^4} - P^4 Y(x) = 0 \quad (2-3)$$

where

$$P^4 = \frac{\mu \omega^2}{EI} \quad (2-4)$$

The general solution of equation (2-3) may be written as

$$Y(x) = Ag_1 + Bg_2 + Cg_3 + Dg_4 \quad (2-5)$$

where



$$g_1 = \frac{1}{2}[\cosh(Px) + \cos(Px)] \quad (a)$$

$$g_2 = \frac{1}{2}[\sinh(Px) - \sin(Px)] \quad (b)$$

(2-6)

$$g_3 = \frac{1}{2}[\cosh(Px) - \cos(Px)] \quad (c)$$

$$g_4 = \frac{1}{2}[\sinh(Px) + \sin(Px)] \quad (d)$$

At any location  $x$ , deflection  $Y_x$ , slope  $Y'_x$ , bending moment  $M_x$ , and shear force  $V_x$  can be expressed as

$$Y_x = Ag_1 + Bg_2 + Cg_3 + Dg_4 \quad (a)$$

$$Y'_x = P(Ag_2 + Bg_3 + Cg_4 + Dg_1) \quad (b)$$

(2-7)

$$M_x = EIP^2(Ag_3 + Bg_4 + Cg_1 + Dg_2) \quad (c)$$

$$V_x = EIP^3(Ag_4 + Bg_1 + Cg_2 + Dg_3) \quad (d)$$

At  $x = 0$ ,

$$Y_0 = A \quad ; \quad Y'_0 = DP \quad (a,b)$$

(2-8)

$$M_0 = C(EIP^2) \quad ; \quad V_0 = B(EIP^3) \quad (c,d)$$

Expressing the constants  $A$ ,  $B$ ,  $C$  and  $D$  in terms of  $Y_0$ ,  $Y'_0$ ,  $M_0$  and  $V_0$  and substituting into equation (2-7) yields:

$$\begin{bmatrix} Y \\ Y' \\ M \\ V \end{bmatrix}_x = \begin{bmatrix} g_1 & g_4/P & g_3/EIP^2 & g_2/EIP^3 \\ Pg_2 & g_1 & g_4/EIP & g_3/EIP^2 \\ EIP^2g_3 & EIPg_2 & g_1 & g_4/P \\ EIP^3g_4 & EIP^2g_3 & Pg_2 & g_1 \end{bmatrix} \begin{bmatrix} Y \\ Y' \\ M \\ V \end{bmatrix}_0 \quad (2-9)$$



The above equation expresses the deflection, slope, bending moment, and shear force at any location  $x$  in terms of those at  $x = 0$ .

## B. TORSION ANALYSIS

The equation of motion of a uniform cylinder undergoing torsional vibration is,

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{\nu}{JG} \frac{\partial^2 \theta}{\partial t^2} = 0 \quad (2-10)$$

where  $\nu$  is mass moment of inertia per unit length,  $J$  is polar moment of inertia of elastic resistance, and  $\theta$  is angle of twist. For a sinusoidal solution at circular frequency  $\omega$ ,

$$\theta(x,t) = \phi(x) \cos(\omega t + \gamma) \quad (2-11)$$

where  $\gamma$  is a constant and  $\phi$  is a function of  $x$  alone. Substitution in equation (2-10) gives

$$\frac{d^2 \phi(x)}{dx^2} + \alpha^2 \phi(x) = 0 \quad (2-12)$$

where

$$\alpha^2 = \frac{\nu \omega^2}{JG} \quad (2-13)$$

The general solution of equation (2-12) is

$$\phi(x) = F \cos(\alpha x) + H \sin(\alpha x) \quad (2-14)$$

and torque  $T$  is

$$T = JG \frac{d\phi(x)}{dx} = JG\alpha [-F \sin(\alpha x) + H \cos(\alpha x)] \quad (2-15)$$



At  $x = 0$

$$\phi_0 = F \quad ; \quad T_0 = H(JG\alpha) \quad (2-16a,b)$$

Expressing constants  $F$  and  $H$  in terms of  $\phi_0$  and  $T_0$  and substituting into equations (2-14) and (2-15), the angle of twist  $\phi_x$  and torque  $T_x$  at any location  $x$ , can be expressed in terms of  $T_0$  and  $\phi_0$  as follows:

$$\begin{bmatrix} \phi \\ T \end{bmatrix}_x = \begin{bmatrix} \cos(\alpha x) & \sin(\alpha x)/JG\alpha \\ -JG\alpha \sin(\alpha x) & \cos(\alpha x) \end{bmatrix} \begin{bmatrix} \phi \\ T \end{bmatrix}_0 \quad (2-17)$$

### C. IN-PLANE AND OUT-OF-PLANE VIBRATION ANALYSIS OF AN ELL-SHAPED UNIFORM FILAMENT WITH CLAMPED ENDS

#### 1. In-plane Vibrations

In Figure 2-1, an ell-shaped configuration and the corresponding free body diagrams are shown.

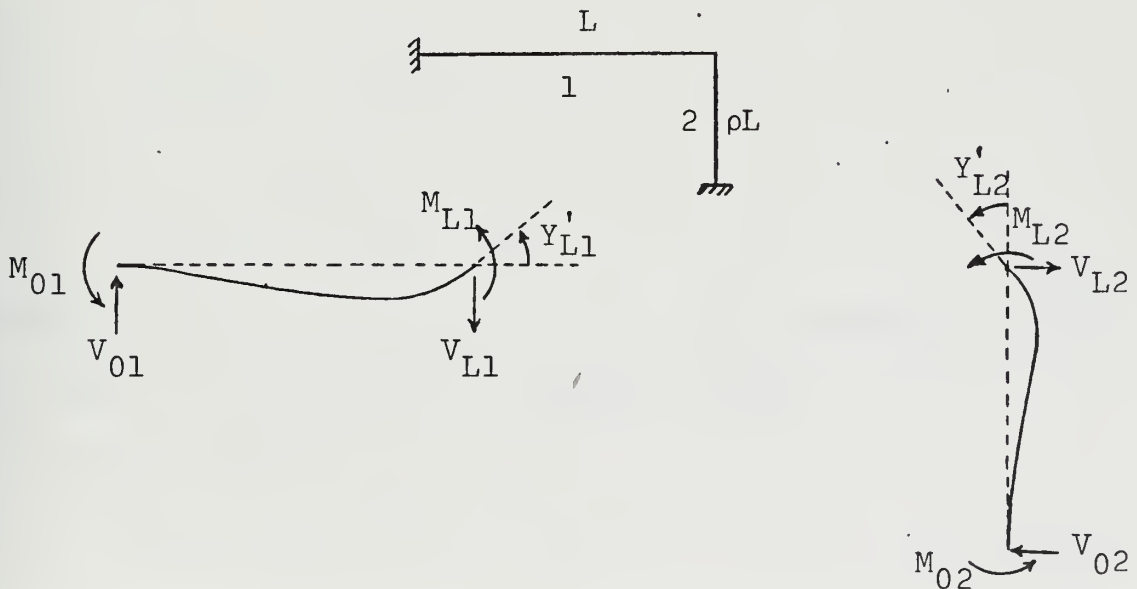


Figure 2-1. Ell-shaped Configuration and Free Body Diagrams for In-plane Vibrations.





The ratio,  $\rho$ , of the length of the shorter leg to that of the longer is a basic geometrical parameter in the following analysis.

At the clamped ends deflections  $Y_{01}$ ,  $Y_{02}$  and slopes  $Y'_{01}$ ,  $Y'_{02}$  are equal to zero. Also at the junction of the two portions, deflections  $Y_{L1}$  and  $Y_{L2}$  are equal to zero. Application of equation (2-9) to the two portions yields,

$$\frac{g_3}{EIP^2} M_{01} + \frac{g_2}{EIP^3} V_{01} = 0 \quad (a)$$

$$\frac{g_4}{EIP} M_{01} + \frac{g_3}{EIP^2} V_{01} = Y'_{L1} \quad (b)$$

$$g_1 M_{01} + \frac{g_4}{P} V_{01} = M_{L1} \quad (c)$$

(2-18)

$$\frac{\bar{g}_3}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^3} V_{02} = 0 \quad (d)$$

$$\frac{\bar{g}_4}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^2} V_{02} = Y'_{L2} \quad (e)$$

$$\bar{g}_1 M_{02} + \frac{\bar{g}_4}{P} V_{02} = M_{L2} \quad (f)$$

where  $g_i$  indicates evaluation of  $g_i(x)$  for argument (PL) and  $\bar{g}_i$  indicates evaluation for argument ( $\rho PL$ ).

At the junction,

$$Y'_{L1} = Y'_{L2} \quad ; \quad M_{L1} + M_{L2} = 0 \quad (2-19a,b)$$

Combining equations (2-19) with equations (2-18), the following set of equations is obtained.



$$\begin{bmatrix} g_3/EIP^2 & 0 & g_2/EIP^3 & 0 \\ 0 & \bar{g}_3/EIP^2 & 0 & \bar{g}_2/EIP^3 \\ g_4/EIP & -\bar{g}_4/EIP & g_3/EIP^2 & -g_3/EIP^2 \\ g_1 & \bar{g}_1 & g_4/P & \bar{g}_4/P \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ V_{01} \\ V_{02} \end{bmatrix} = 0 \quad (2-20)$$

For a nontrivial solution, the determinant, D, of the above matrix must vanish. Elementary manipulations on D lead to the following form.

$$\begin{vmatrix} g_3 & 0 & g_2 & 0 \\ 0 & \bar{g}_3 & 0 & \bar{g}_2 \\ g_4 & -\bar{g}_4 & g_3 & -\bar{g}_3 \\ g_1 & \bar{g}_1 & g_4 & \bar{g}_4 \end{vmatrix} = 0 \quad (2-21)$$

The terms of equation (2-21) are functions of  $\rho$ . For any value of  $\rho$ , the zeros  $(PL)_i$  of the determinant can be found. The corresponding frequencies are evaluated by using equation (2-4) as

$$\omega_i = (PL)_i^2 \sqrt{\frac{EI}{\mu L^4}} \quad (2-22)$$

Solution of equation (2-21) will be explained in Section III-A.

## 2. Out-of-plane Vibrations

Out-of-plane vibrations can be analyzed similarly. However in this case the effect of torsion must be taken into account.



In Figure 2-2, free body diagrams of an ell-shaped configuration for out-of-plane vibrations are shown.

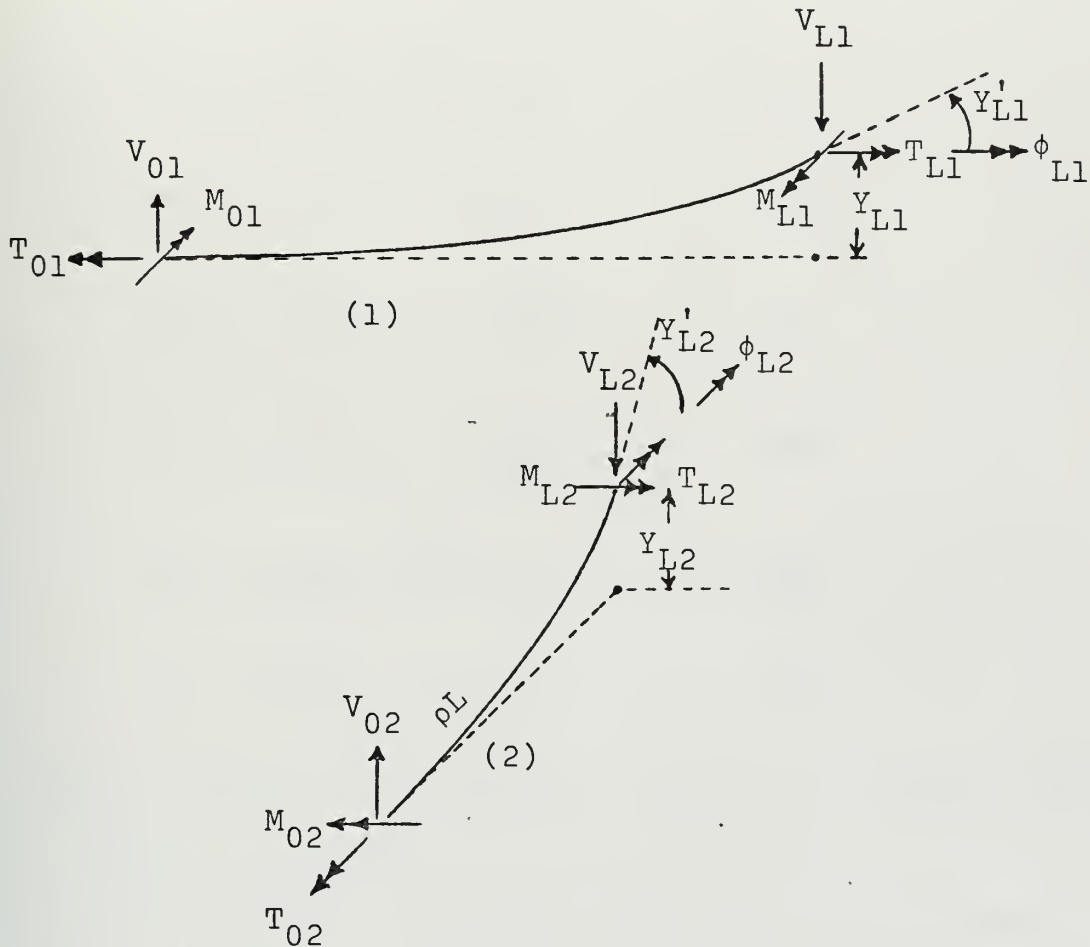


Figure 2-2. Free Body Diagrams of an Ell-shaped Configuration for Out-of-plane Vibrations.

At the clamped ends deflections  $Y_{01}$ ,  $Y_{02}$  and slopes  $Y'_{01}$ ,  $Y'_{02}$  and angles of twist  $\phi_{01}$ ,  $\phi_{02}$  are equal to zero. Application of equation (2-9) to the two portions yields



$$Y_{L1} = \frac{g_3}{EIP^2} M_{01} + \frac{g_2}{EIP^3} V_{01} \quad (a)$$

$$Y'_{L1} = \frac{g_4}{EIP} M_{01} + \frac{g_3}{EIP^2} V_{01} \quad (b)$$

$$M_{L1} = g_1 M_{01} + \frac{g_4}{P} V_{01} \quad (c)$$

$$V_{L1} = P g_2 M_{01} + g_1 V_{01} \quad (d)$$

$$Y_{L2} = \frac{\bar{g}_3}{EIP^2} M_{02} + \frac{\bar{g}_2}{EIP^3} V_{02} \quad (e) \quad (2-23)$$

$$Y'_{L2} = \frac{\bar{g}_4}{EIP} M_{02} + \frac{\bar{g}_3}{EIP^2} V_{02} \quad (f)$$

$$M_{L2} = \bar{g}_1 M_{02} + \frac{\bar{g}_4}{P} V_{02} \quad (g)$$

$$V_{L2} = P \bar{g}_2 M_{02} + \bar{g}_1 V_{02} \quad (h)$$

where  $g_i$  indicates evaluation of  $g_i(x)$  for argument (PL) and  $\bar{g}_i$  indicates evaluation for argument ( $\rho$ PL).

For the torsion, equation (2-17) applies and gives,

$$\phi_{L1} = \frac{\sin(\alpha L)}{JG\alpha} T_{01} \quad (a)$$

$$T_{L1} = \cos(\alpha L) T_{01} \quad (b)$$

$$\phi_{L2} = \frac{\sin(\alpha \rho L)}{JG\alpha} T_{02} \quad (c)$$

$$T_{L2} = \cos(\alpha \rho L) T_{02} \quad (d)$$

At the junction:





$$M_{L1} = T_{L2} \quad ; \quad T_{L1} = -M_{L2} \quad (a,b)$$

$$Y'_{L1} = -\phi_{L2} \quad ; \quad \phi_{L1} = Y'_{L2} \quad (c,d) \quad (2-25)$$

$$V_{L1} = -V_{L2} \quad ; \quad Y_{L1} = Y_{L2} \quad (e,f)$$

Specializing to the case of circular cross section and assuming Poisson's ratio is 0.3, then

$$\frac{1}{JG\alpha} = \frac{1.3}{EI\alpha} \quad (2-26)$$

Combination of equations (2-23), (2-24), (2-25) and (2-26) gives the following set of equations.

$$\begin{bmatrix} g_3 P & -\bar{g}_3 P & 0 & 0 & g_2 & -\bar{g}_2 \\ 0 & \bar{g}_4 P & -\frac{1.3P^2}{\alpha} \sin(\alpha L) & 0 & 0 & \bar{g}_3 \\ g_4 P & 0 & 0 & \frac{1.3P^2}{\alpha} \sin(\alpha L) & g_3 & 0 \\ g_2 P & \bar{g}_2 P & 0 & 0 & g_1 & \bar{g}_1 \\ g_1 & 0 & 0 & -\cos(\alpha L) & \frac{g_4}{P} & 0 \\ 0 & \bar{g}_1 & \cos(\alpha L) & 0 & 0 & \frac{\bar{g}_4}{P} \end{bmatrix} \begin{bmatrix} M_{01} \\ M_{02} \\ T_{01} \\ T_{02} \\ V_{01} \\ V_{02} \end{bmatrix} = 0 \quad (2-27)$$

For a nontrivial solution, the determinant D of this matrix must vanish. From equations (2-4), (2-13) and (2-26) it is found that

$$(\alpha L) = \beta(PL)^2 \quad (2-28)$$



where

$$\beta = \sqrt{\frac{1.3v}{\mu L^2}} \quad (2-29)$$

is a dimensionless parameter. Substituting equation (2-28) into equation (2-27) and performing elementary manipulations on D give the following equation

$$\begin{vmatrix} g_3 & -\bar{g}_3 & 0 & 0 & g_2 & -\bar{g}_2 \\ 0 & \bar{g}_4 & -\frac{1.3}{\beta} \sin[\beta(PL)^2] & 0 & 0 & g_3 \\ g_4 & 0 & 0 & \frac{1.3}{\beta} \sin[\rho\beta(PL)^2] & g_3 & 0 \\ g_2 & \bar{g}_2 & 0 & 0 & g_1 & \bar{g}_1 \\ g_1 & 0 & 0 & (PL)\cos[\beta\rho(PL)^2] & g_4 & 0 \\ 0 & \bar{g}_1 & (PL)\cos[\beta(PL)^2] & 0 & 0 & \bar{g}_4 \end{vmatrix} = 0 \quad (2-30)$$

The terms of equation (2-30) are functions of  $\rho$  and  $\beta$ . For given values of  $\rho$  and  $\beta$ , the zeros  $(PL)_i$  of determinant D can be found. The corresponding frequencies are evaluated by using equation (2-22).

Solution of equation (2-30) will be explained in Section III-A.



### III. PRESENTATION OF RESULTS

#### A. METHODS OF SOLUTION

The equations (2-21) and (2-30), developed in Chapter II, can be solved either by algebraic manipulations or by using direct computer solution.

In a first approach to a solution, algebraic manipulations were performed to reduce the order of determinants. Then transcendental equations were obtained and two computer programs, one for the in-plane solution and one for the out-of-plane solution, were developed to get the roots of these equations.

In the computer programs the transcendental equations were used as function subprograms and roots located in an interval. Then the Newton-Raphson method was used to obtain good approximate values of the roots.

The procedure explained above led to some difficulties. During the solution of the equation which was derived from equation (2-30), the Newton-Raphson method failed to give an expected root because the slope of the transcendental function reached very high values and this caused a jump to adjacent roots. In one case, the number of iterations required exceeded three hundred with an allowable error of  $10^{-8}$  (for the quantity PL) and increasing the allowable error to  $10^{-6}$  did not make much difference. Finally, comparison of results with the classical solution in the case  $\rho$  is



equal to zero showed that some of the roots were extraneous.

The difficulties mentioned in the preceding paragraph were eliminated by making direct computer evaluation of the determinants (2-21) and (2-30). For this purpose subroutine BRODE was developed and each element of the determinants was stored in this subprogram. In addition to subroutine BRODE, library subroutine DTERM which was developed by Jean Bow in 1967 was used to evaluate the determinants. This modified procedure resulted in reduction of computer time required and in the elimination of extraneous roots.

Difficulties with occasional failure of the Newton-Raphson method were reduced by using the method of successive bisectioning. In this method the number of iterations required for one root did not exceed fifty with an allowable error of  $10^{-6}$ . This procedure also resulted in conservation of computer time.

Detailed instructions for use of the computer programs are given in Appendix B.

## B. GRAPHS

Numerical results given to a large number of significant figures were obtained primarily for the purpose of investigating the accuracy and integrity of the solutions. Chapter IV will give a discussion of these investigations.

However, so as to present the results in the most convenient form for practical use, graphical outputs





were also obtained by use of the library subroutine DRAW.

One graph was obtained for the in-plane solution. For the out-of-plane solution five graphs were obtained corresponding to five different values of  $\beta$ . In each case, the ordinates are values of (PL) and abscissas are values of  $\rho$ . On these graphs the first (lowest) curve is for the first mode, the second curve for the second mode and so on. Graphs are given in Figures 3-1 through 3-6.

### C. SAMPLE PROBLEM

A sample problem will be solved to show how to use the graphs given in the preceding article.

Properties of ell-shaped configuration:

$L_1$	:	16' - 6"
$L_2$	:	10' - 3½"
Outside diameter of pipe	:	6.625"
Inside diameter of pipe	:	6.065"
Wall thickness of pipe	:	0.280"
Weight of pipe	:	18.98 lb/ft
Thickness of insulation	:	1 1/8"
Weight of insulation	:	2.90 lb/ft
Weight of contents	:	12.51 lb/ft
Modulus of elasticity E	:	27,400,000 lb/in <sup>2</sup>
G	:	10,600,000 lb/in <sup>2</sup>

For the use of graphs, values of  $\rho$  and  $\beta$  must be known.  
For this problem,



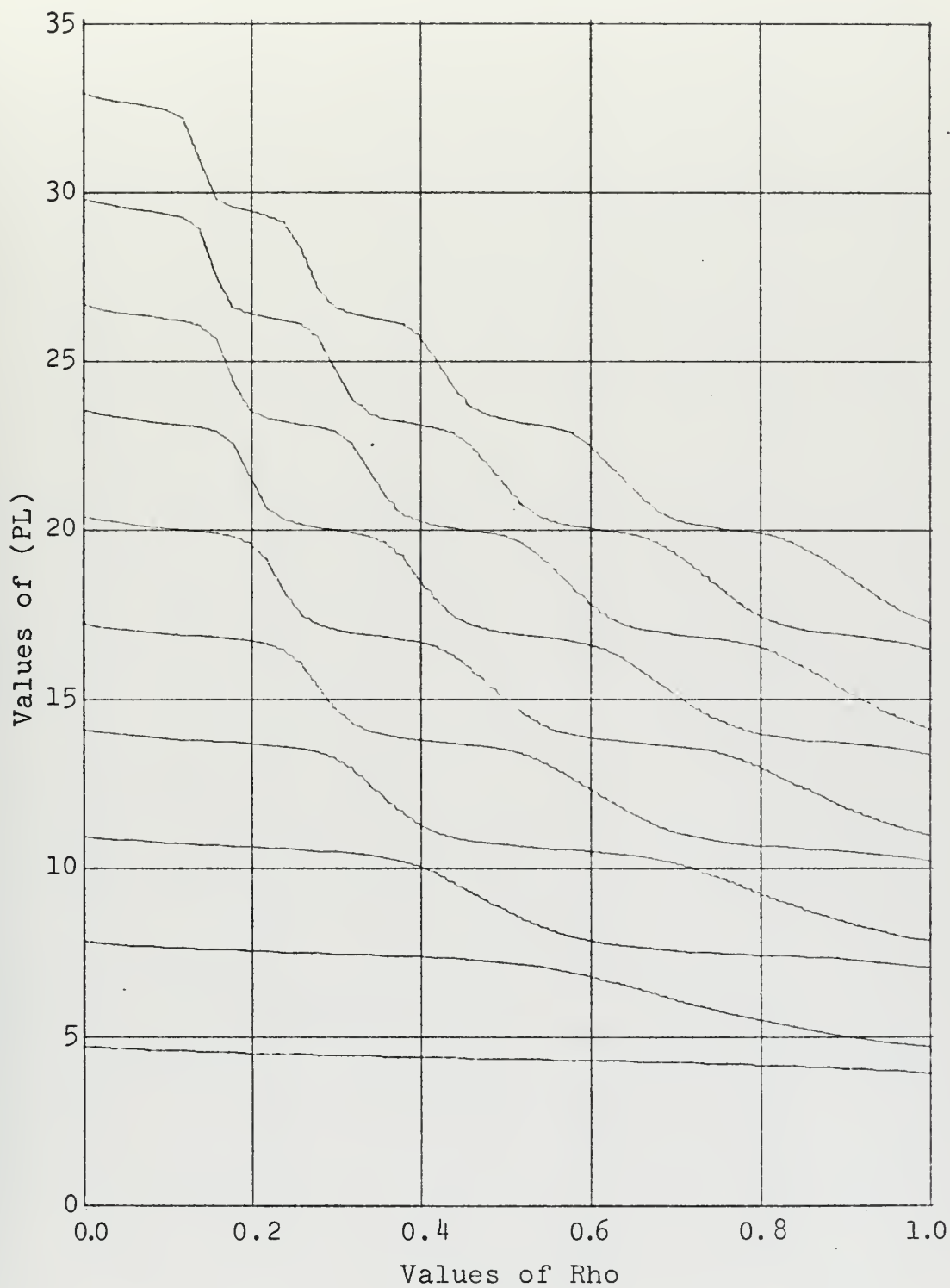


Figure 3-1. In-plane Vibrations.



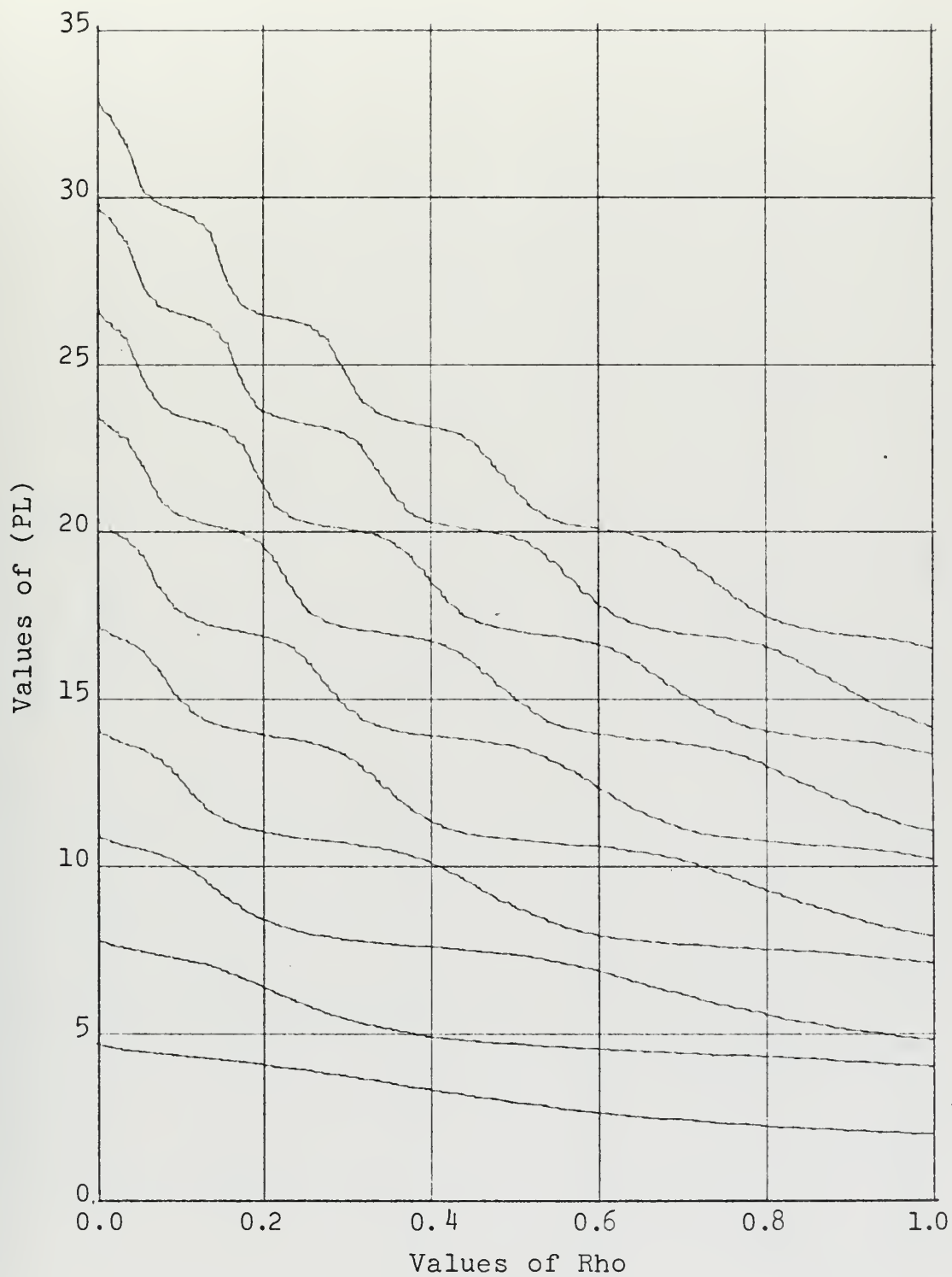


Figure 3-2. Out-of-plane Vibrations, Beta = 0.00001.



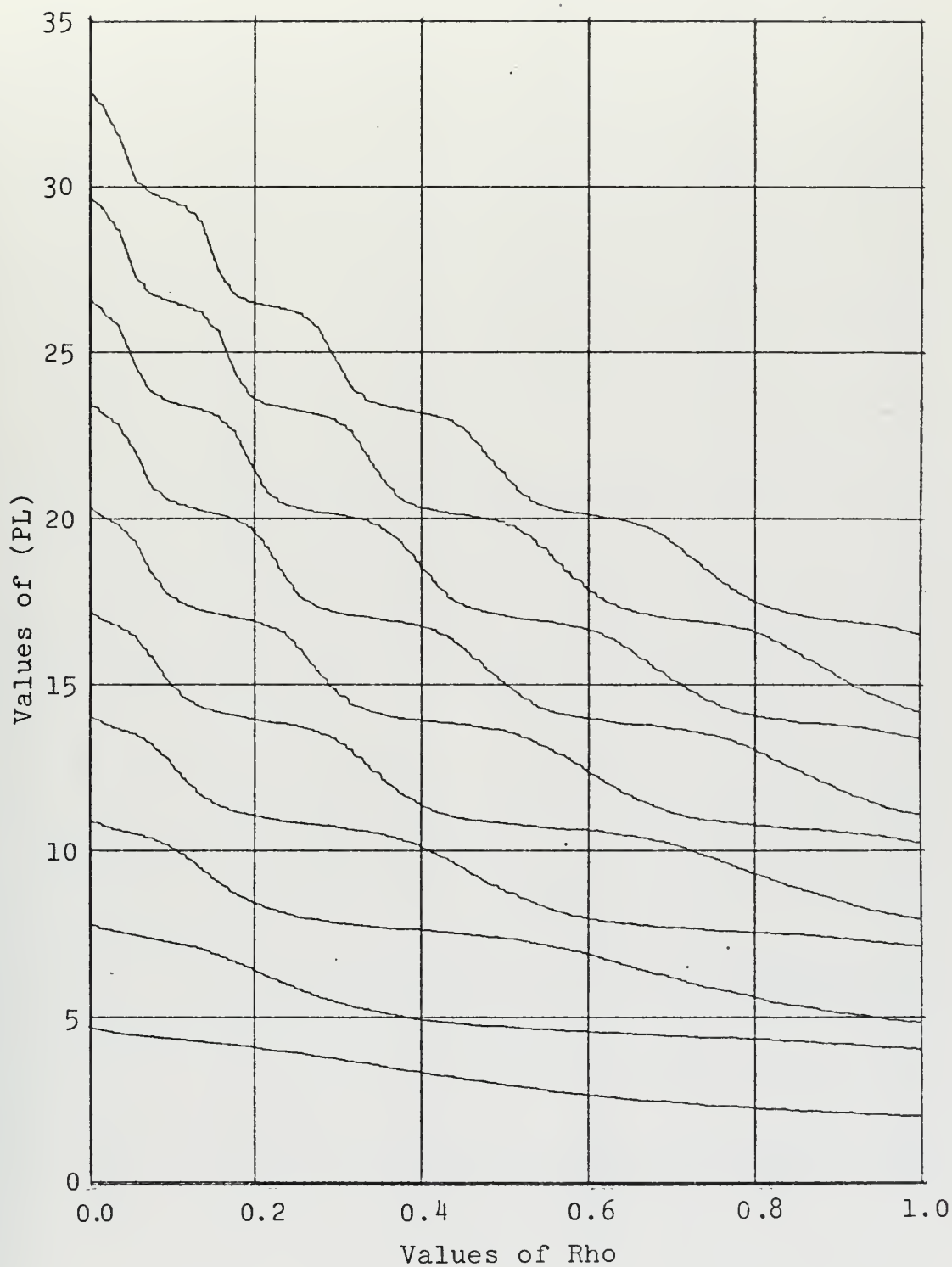


Figure 3-3. Out-of-plane Vibrations,  $\beta = 0.001$ .





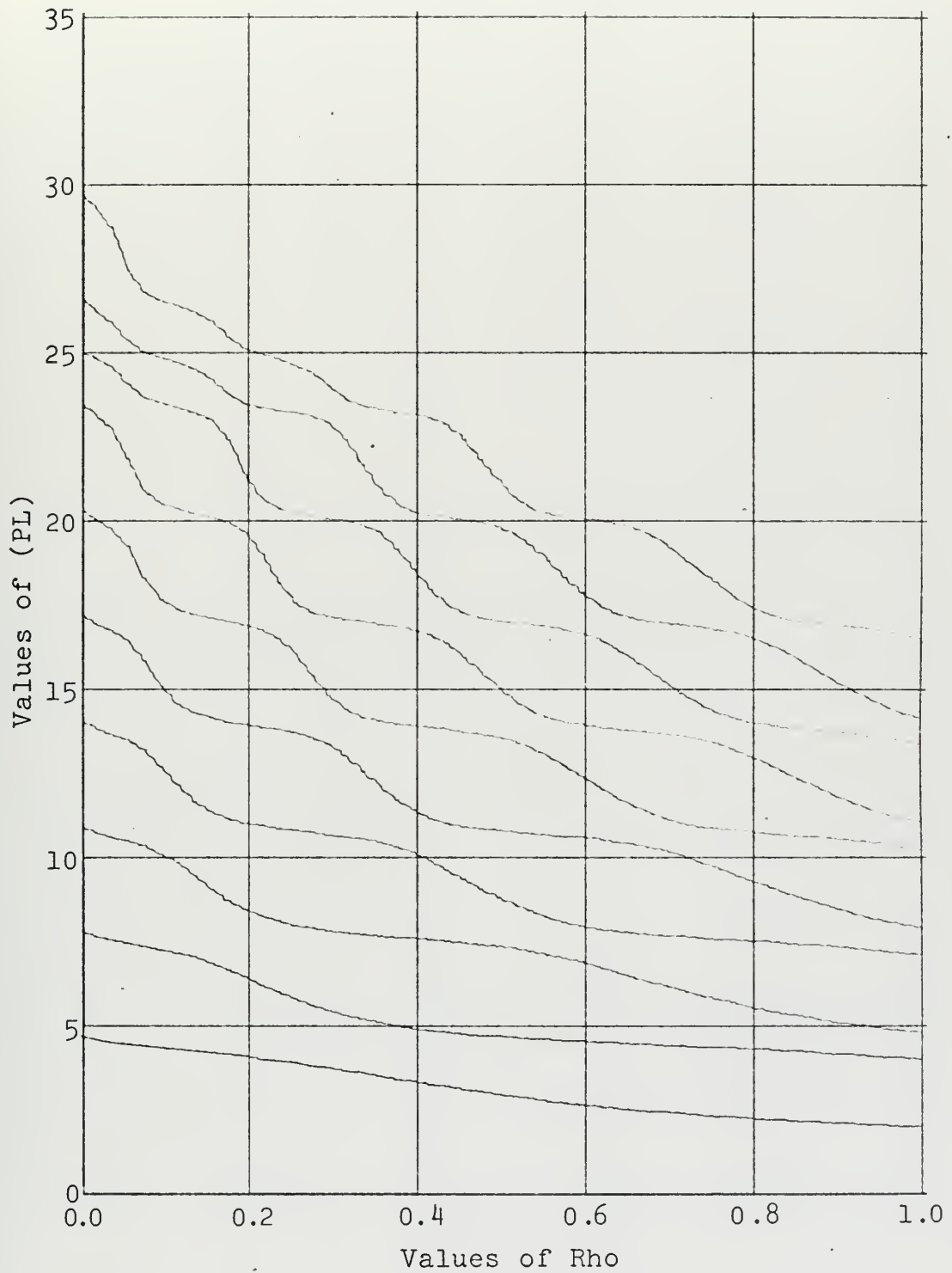


Figure 3-4. Out-of-plane Vibrations,  $\beta = 0.005$ .



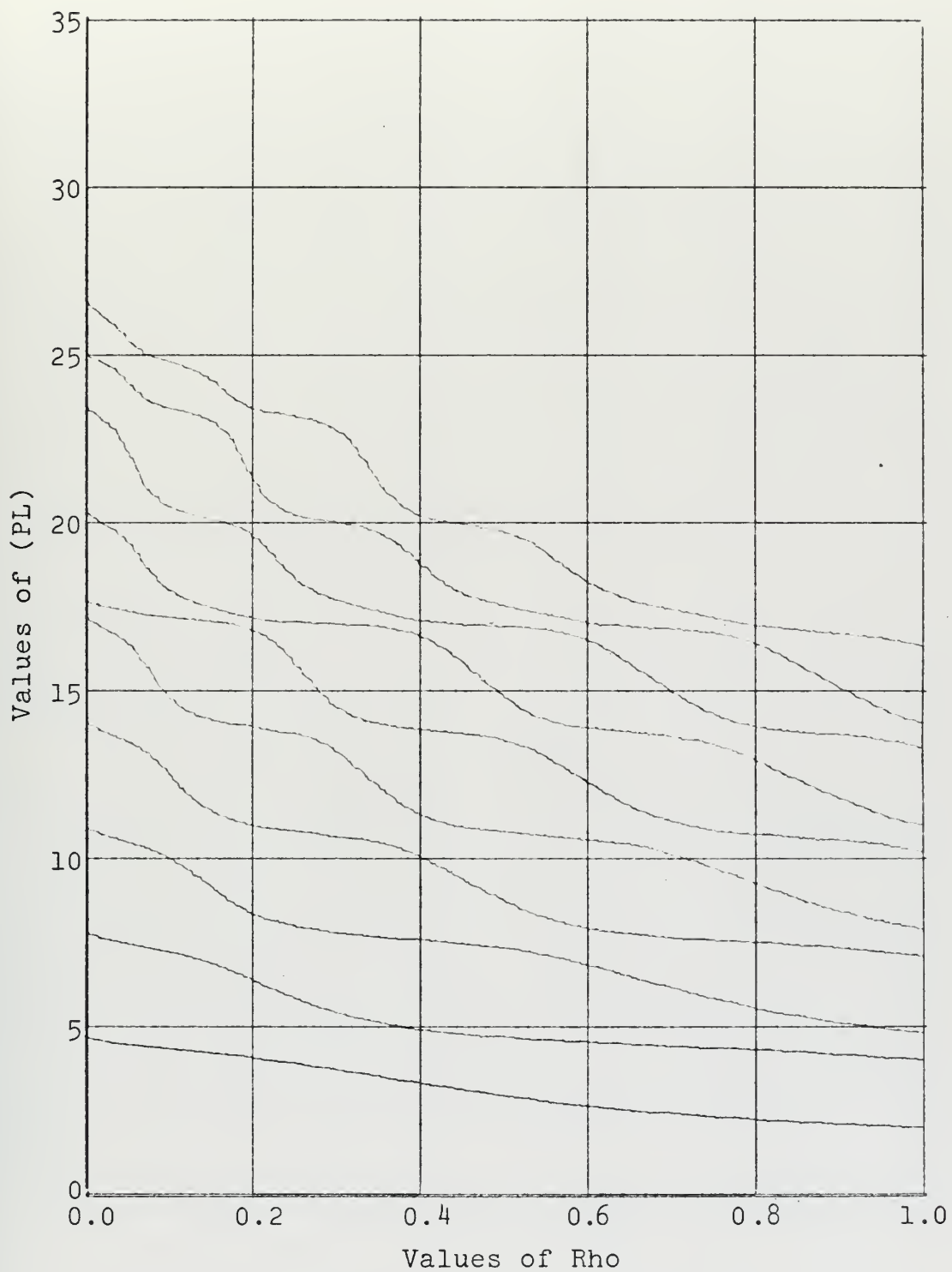


Figure 3-5. Out-of-plane Vibrations,  $\beta = 0.01$ .



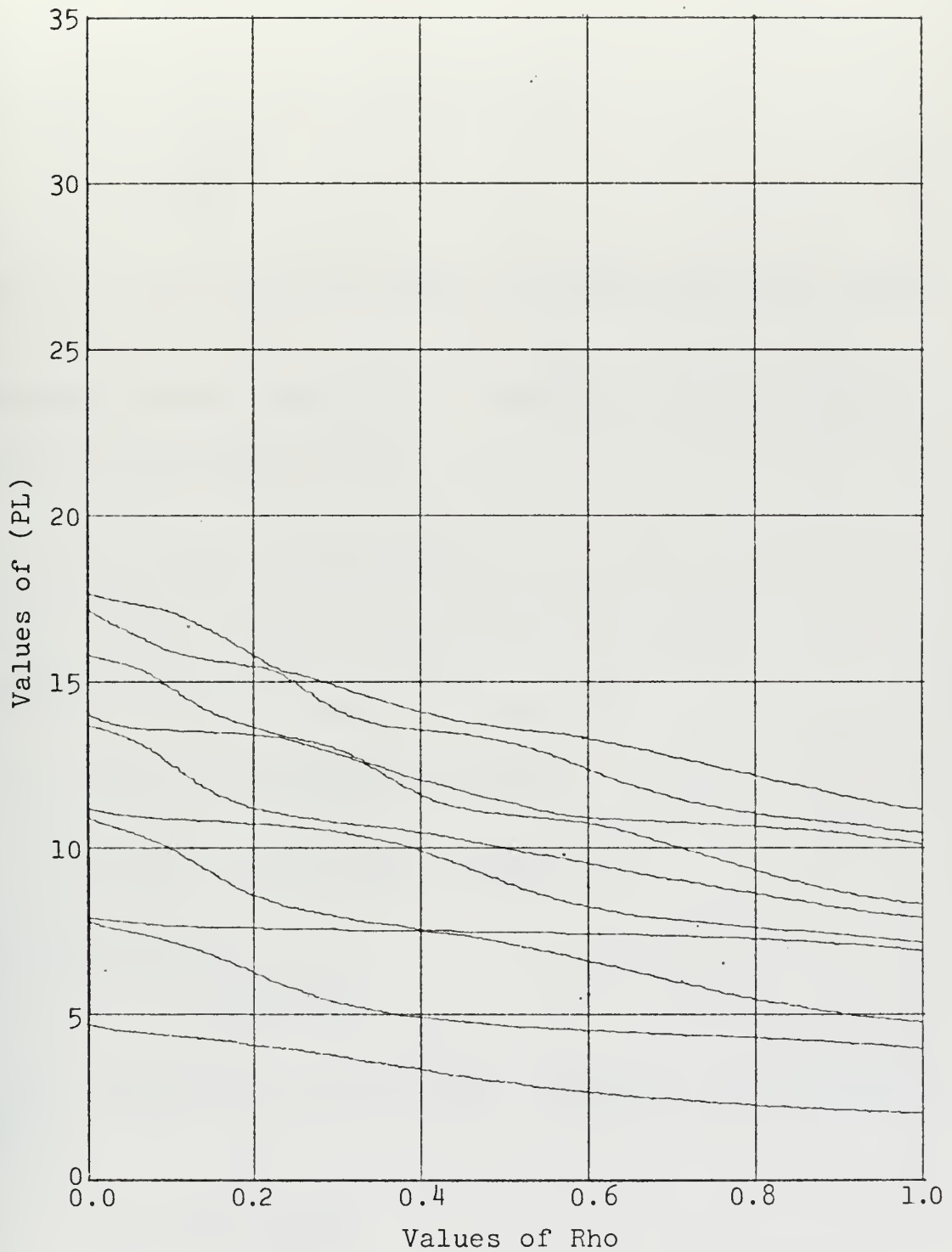


Figure 3-6. Out-of-plane Vibrations,  $\beta = 0.05$ .



$$\rho = \frac{L_2}{L_1} = 0.62373737\ldots$$

and  $\beta$  can be found from equation (2-29).

$$\beta = \sqrt{\frac{1.3v}{\mu L_1^2}} \quad (3-1)$$

where  $v$  is the total mass moment of inertia per unit length for the torsion and  $\mu$  is the total mass per unit length undergoing lateral vibrations. Assuming that the contents (water) do not rotate, then

$$v = v_{\text{pipe}} + v_{\text{insulation}}$$

$$\mu_{\text{total}} = \mu_{\text{pipe}} + \mu_{\text{insulation}} + \mu_{\text{contents}}$$

numerical values of  $v_{\text{total}}$  and  $\mu_{\text{total}}$  are

$$v_{\text{total}} = 0.0509089405 \text{ lb-sec}^2$$

$$\mu_{\text{total}} = 1.068875490 \text{ lb-sec}^2\text{-in}^{-2}$$

and from equation (3-1)

$$\beta = 0.0150806896$$

For evaluation of frequencies, equation (2-22) should be used as

$$\omega_i = (PL)_i^2 \sqrt{\frac{EI}{\mu L_1^4}}$$

then

$$\sqrt{\frac{EI}{\mu L_1^4}} = 8.221317961 \text{ sec}^{-1}$$





To solve the problem, values of  $(PL)_1$  can be found from the graphs. But no graph is available for the value of  $\beta$  found above. Therefore interpolation is required between the values taken from Figure 3-5 and from Figure 3-6. Values of  $(PL)_1$  are given in the table below where the exact solution was obtained by a computer solution using the specified value for  $\beta$ .

MODE NO.	EXACT SOLUTION	SOLUTION OBTAINED BY LINEAR INTERPOLATION
1	2.624579	2.624302
2	4.533513	4.521991
3	6.729044	6.706905
4	7.862686	7.817582
5	10.518163	10.230722
6	11.908733	11.698410
7	13.730277	13.442683
8	14.259158	15.619057
9	16.376500	16.379861
10	17.185014	17.403866

As seen from the table, for the lower modes, linear interpolation gives good results. However there is greater error in using linear interpolation for the higher frequencies. One can make what appears to be the reasonable conjecture that if one were to plot curves with  $(PL)$  as ordinate and  $\beta$  as abscissa and with different curve sheets corresponding to



fixed values of  $\rho$ , there might be some crossing of individual curves; that is, for certain values of  $\beta$  two distinct modes might have the same frequency. Clearly this would confuse the significance of characterizing different modes simply by ordinal number in the order of increasing frequency and would indicate that a more sophisticated interpolation is required. However, no such graphs were prepared and there do not seem to be enough values of  $\beta$  represented in the graphs at hand to permit making replots. Thus it is suggested that some future investigator might find it interesting to employ the computer programs, incorporating slight modifications, to obtain such graphs and thus investigate the reasons for inaccurate interpolation for the higher modes.



#### IV. ACCURACY AND ASSURANCE OF SOLUTION

The allowable error permitted in determining the roots is  $10^{-6}$  and this was assumed to be satisfactory for practical purposes. The allowable error can be changed from  $10^{-6}$  to any desired value by changing the value EPS in the computer programs. This can be done by changing the card 000090 in the program IN-PLANE and card 000100 in the program OUT-OF-PLANE.

Assurance of solution was established in the following ways.

1. In the case  $\rho$  is equal to zero, the ell-shaped configuration becomes a simple built-in beam and solution for this case can be compared with the classical solution.

Degenerate Case; $\rho = 0$			
Mode No.	Values of (PL)		
	Comparison Values*	In - plane Solution	Out-of-plane Solution
1	4.7300407	4.7300289	4.7300411
2	7.8532046	7.8531849	7.8532047
3	10.9956078	10.9955803	10.9956081
4	14.1371655	14.1371302	14.1371655
5	17.2787597	17.2787165	17.2787594
6	20.4203522	20.4203012	20.4203526
7	23.5619449	23.5618858	23.5619449
8	26.7035376	26.7034814	26.7035412
9	29.8451302	29.8453738	29.8451861
10	32.9867229	32.9890104	32.9866973

\* Obtained by computer solution of the equation  $\cosh(PL) \cos(PL) = 1$ .



In equation (2-21) when  $\rho$  is set to zero the value of the determinant is zero regardless of the value of  $(PL)$ . Therefore  $\rho$  was taken to be 0.00001.

In out-of-plane case for the larger values of  $\beta$ , pure torsional modes appeared and this can be seen from Figures (3-5) and (3-6). The values of  $(PL)$  given in the above table were taken when  $\beta$  is equal to 0.001.

2. Consider the cases shown in Figure 4-1.

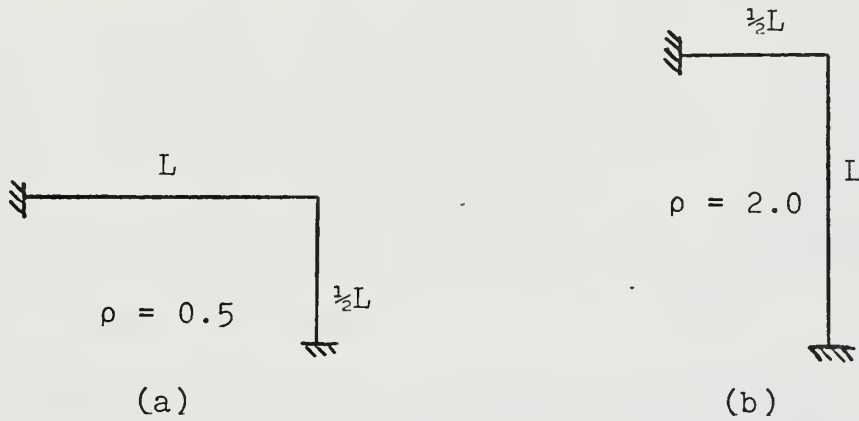


Figure 4-1. Cases Having Reciprocal Values of  $\rho$ .

The configurations shown in the above figure are identical, therefore two systems must have the same frequencies. Using equation (2-22) as

$$(PL)_{ai}^2 \sqrt{\frac{EI}{\mu L^4}} = (PL)_{bi}^2 \sqrt{\frac{EI}{\mu (\frac{L}{2})^4}}$$

then

$$\frac{(PL)_{ai}}{(PL)_{bi}} = 2$$





Mode No.	$\rho = 0.5$	$\rho = 2.0$	$\frac{(PL)_{ai}}{(PL)_{bi}}$
	$(PL)_{ai}$	$(PL)_{bi}$	
1	4.3755336	2.1877668	2.000000000
2	7.2490132	3.6245066	2.000000000
3	8.7635958	4.3817979	2.000000000
4	10.707676	5.3538381	1.999999999
5	13.540341	6.7701705	2.000000001
6	15.024013	7.5120064	1.999999999
7	16.987974	8.4939872	2.000000000
8	19.8231918	9.9115959	2.000000000
9	21.3082077	10.6541038	2.000000001
10	23.2712829	11.6356415	2.000000001

For out-of-plane solution in the case  $\beta = 0.001$

Mode No.	$\rho = 0.5$	$\rho = 2.0$	$\frac{(PL)_{ai}}{(PL)_{bi}}$
	$(PL)_{ai}$	$(PL)_{bi}$	
1	2.9832348	1.4916187	1.999998209
2	4.7014996	2.3507534	1.999996916
3	7.3772076	3.6886158	1.999993485
4	8.8416393	4.4208958	1.999965571
5	10.8159008	5.4080044	1.999980037
6	13.6035633	6.8018597	1.999977062
7	15.0968311	7.5488148	1.999894233
8	17.0625606	8.5315098	1.999946210
9	19.8673419	9.9339245	1.999948965
10	21.3568783	10.6796268	1.999777575



3. In the case  $\rho$  is equal to one, it is expected that among the in-plane solutions will be many for which the moment at the joint vanishes. Thus, these cases also pertain to the clamped-pinned simple beam. Now two clamped-pinned simple beams of different lengths,  $\ell$  and  $\ell^*$ , may have matching frequencies in their spectra. That is, for example, the second frequency for one may equal the first frequency for the other. Specifically the following case was considered.

The third frequency for an equal-leg ell is equal to the second frequency of a clamped-pinned beam of length  $\ell$ , namely

$$\omega_3 = (PL)_3^2 \sqrt{\frac{EI}{\mu \ell^4}}$$

where

$$(PL)_3 = 7.06858275$$

The first frequency of an equal-leg ell of length  $\ell^*$  is equal to the first frequency of clamped-pinned beam of length  $\ell^*$ , namely

$$\omega_1 = (PL)_1^2 \sqrt{\frac{EI}{\mu \ell^{*4}}}$$

where

$$(PL)_1 = 3.92660231$$

The ratio  $\rho$  was determined so that then the frequencies are equal, thus



$$\rho = \frac{\ell^*}{\ell} = \frac{(PL)_1}{(PL)_3} = 0.55550065$$

Then performing an analysis for an ell-shaped configuration with the value of  $\rho$ , found above, the second frequency is indeed,

$$\omega_2 = (PL)_2 \sqrt{\frac{EI}{\mu \ell^4}}$$

where

$$(PL)_2 = 7.068384686$$

This value shows close agreement with the value

$$(PL)_3 = 7.0685275$$

given above.



## APPENDIX A

### DESCRIPTION OF PROGRAMS

#### A. GENERAL REMARKS

Programs IN-PLANE and OUT-OF-PLANE are FORTRAN-IV language digital computer programs designed for use in the vibration analysis of an ell-shaped uniform filament with clamped ends.

Both programs employ double precision arithmetic. Subroutines are provided in binary form. Trigometric and hyperbolic functions, available in the IBM-360/67 library, were used.

#### B. PROGRAM STRUCTURE

Each program consists of a four section main program and two subroutines. The function of each section of the main program and of each subroutine is given below.

##### 1. Main program subsections and their functions.

INPUT	Controls read-in of data and allocates storage locations for arrays and subscripted variables. The allowable error for determination of roots is specified in this section.
-------	---

ITERATION AND EVALUATION OF ROOTS	In this section the roots of the determinant are located in an interval by searching the sign changes of the determinant and then the method of successive bisectioning is used to evaluate the roots. The maximum number of iterations was limited to three hundred and this was assumed sufficient. If the number of iterations becomes as large as three
--	---





hundred the process terminates and the approximate root is tagged to indicate a possible inaccurate result.

OUTPUT Controls output format.

GRAPH Controls the graph output and uses the DRAW subroutine provided in the IBM-360/67 library.

## 2. Subroutines and their functions.

BRODE In program IN-PLANE equation (2-21) is stored and in program OUT-OF-PLANE equation (2-30) is stored. Evaluates each element of the determinant and obtains the value of the determinant by calling subroutine DTERM.

DTERM This subroutine computes the determinant of a matrix of real numbers by Gauss' method of elimination with row pivoting.

## C. PROGRAM NOMENCLATURE

A Elements of determinants.

B Checks sign of determinant and if there is a sign change moves control to the method of successive bisectioning.

BB Values of  $\beta$  in subroutine BRODE.

BBB Checks the acceptability of roots.

BETA Value of  $\beta$ .

CEM Single precision version of R0.

DET Value of determinant.

EPS Allowable error in roots.

F Value of determinant.

N Index of modes (Ordinal number).

P Values of (PL) used to search interval.

R3, R4 Values of determinant in the method of successive bisectioning.

R0 Values of  $\rho$ .



ROOT	Roots of determinant (PL).
RR	Corresponds to RO in subroutine BRODE.
SEV	Single precision version of ROOT.
X	Middle of interval.
XX1; XX2	Upper and lower limits of interval containing the root.
YY	Values of P, X, XX1 and XX2 in subroutine BRODE.



## APPENDIX B

### INSTRUCTIONS FOR USE OF PROGRAMS

#### A. DATA REQUIRED

##### 1. Data required for program IN-PLANE

- a. Number of  $\rho$ 's (NRO).
- b. Number of modes sought (MODE).
- c. Graph control (NGRAPH).
- d. Values of  $\rho$  (RO).
- e. Graph title (ITITLE).

##### 2. Data required for program OUT-OF-PLANE

- a. Number of  $\rho$ 's (NRO)
- b. Number of  $\beta$ 's (NBETA).
- c. Number of modes sought (MODE).
- d. Graph control (NGRAPH).
- e. Values of  $\beta$  (BETA).
- f. Values of  $\rho$  (RO).
- g. Graph title (ITITLE).

#### B. DATA DECK ASSEMBLY

##### 1. Program IN-PLANE

The data deck is ordered as follows:

- a. Problem statement card.
- b. Values of  $\rho$  card(s).
- c. Graph title cards.

##### 2. Program OUT-OF-PLANE



The data deck is ordered as follows:

- a. Problem statement card.
- b. Values of  $\beta$  card(s).
- c. Values of  $\rho$  card(s).
- d. Graph title cards.

#### C. LIMITING THE NUMBER OF ITERATIONS

In the programs the maximum number of iterations was set to 300. If the number of iterations reaches 300 for any root, the current value is assumed as a solution but in the output it is marked as ITER. EXCEEDED 300. A user either accepts the root or changes the cards 000430 in program IN-PLANE and 000460 in program OUT-OF-PLANE by changing 300 to any desired value.

#### D. GRAPH

A user can get graphical output by option. DRAW subroutine, available in IBM-360/67 library was used and three CALL DRAW statements control first, intermediate and last curves to be plotted on one graph. DRAW subroutine requires single precision arithmetic. Therefore variables CEM and SEV are introduced to change the values of RO and ROOT from double precision to single precision.

#### E. FORMAT OF DATA

##### 1. Program IN-PLANE

- a. Problem statement card.

The card has 3 fields under the control of following field specification.





Field names and the information each field conveys are given below in order.

Field names and content.

- (1) NRO            Number of  $\rho$ 's
- (2) MODE          Number of modes sought.
- (3) NGRAPH       Code specifying whether there is  
                  to be graph output.  
                  (NGRAPH = 0   No graph output)  
                  (NGRAPH = 1   Graph will be output)

b. Values of  $\rho$  card(s).

Field specification 7F10.0.

If there are more than 7 different values of  $\rho$ , additional card(s) will be required with the same field specification.

c. Graph title cards.

The title of graph is read as data with the field specification 6A8. For the title two cards are required. The first card corresponds to the first line of the title and the second card corresponds to the second line.

## 2. Program OUT-OF-PLANE

a. Problem statement card.

The card has 4 fields under the following field specification

Field names are given below in order.



- (1) NRO            Number of  $\rho$ 's.
- (2) NBETA        Number of  $\beta$ 's.
- (3) MODE        Number of modes sought.
- (4) NGRAPH      Same as in program IN-PLANE

b. Values of  $\beta$  cards.

Field specification 7F10.0 and field name is BETA.

c. Values of  $\rho$  card(s).

Same as in program IN-PLANE.

d. Graph title cards.

Same as in program IN-PLANE.



## APPENDIX C

### FLOW DIAGRAMS

#### A. GENERAL

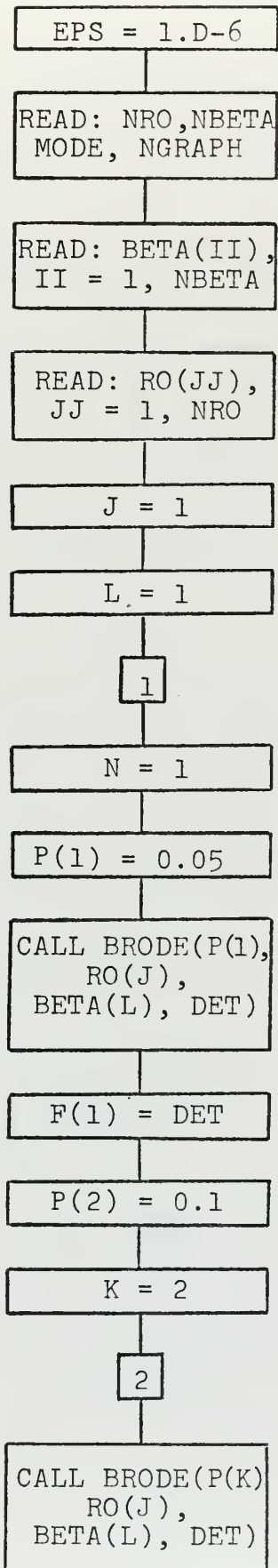
Flow diagram of the program OUT-OF-PLANE is given. This flow diagram includes all parts of the main program. Flow diagrams of subroutines BRODE and DTERM are not given and their functions were explained in Section A-2 in Appendix A.

Program IN-PLANE is similar to OUT-OF-PLANE, therefore its flow diagram is not given.

#### B. FLOW DIAGRAM OF PROGRAM OUT-OF-PLANE

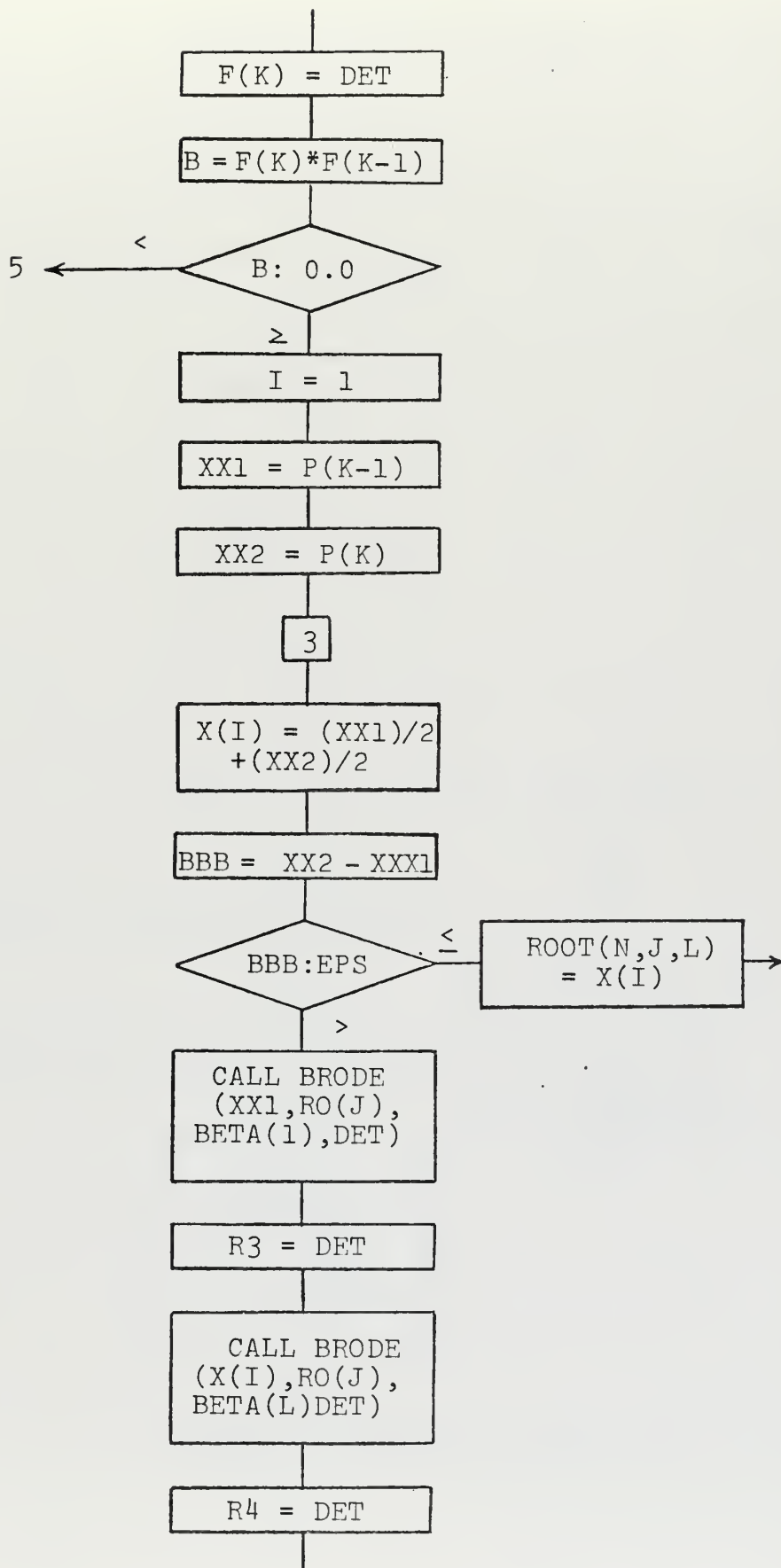
Flow diagram follows.



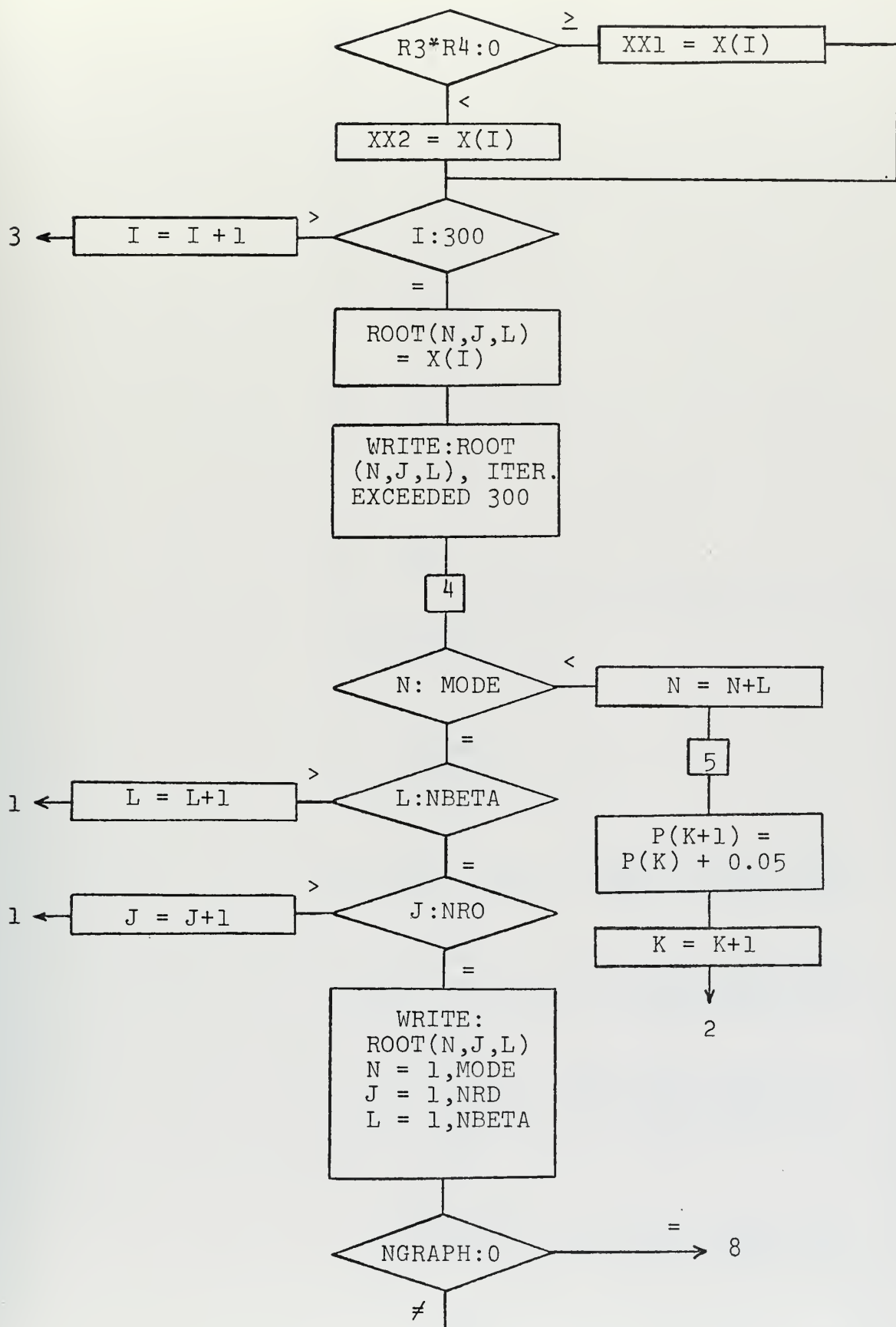




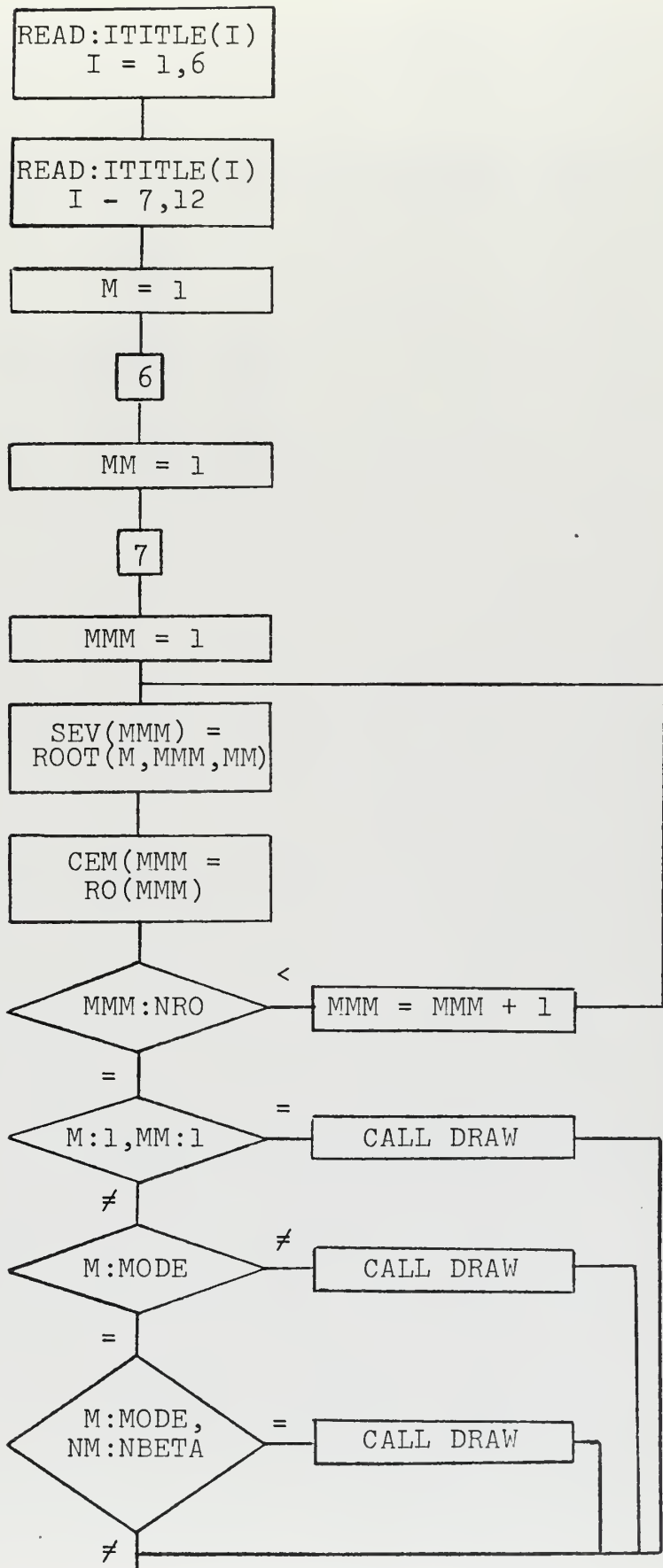




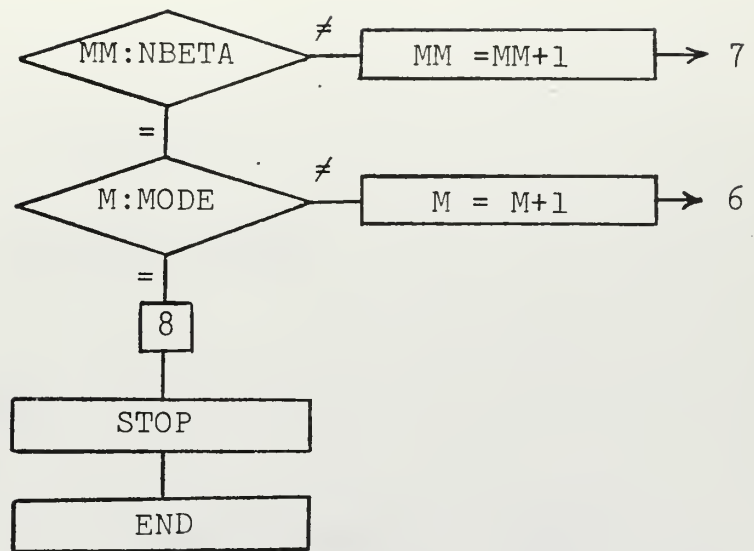
















## APPENDIX D

### PROGRAM LISTING

#### A. INDEX

##### 1. Program IN-PLANE

Main program:

<u>Section</u>	<u>Sequence number</u>	<u>Page</u>
INPUT	000090 - 000130	53
ITERATION AND EVALUATION OF ROOTS	000170 - 000550	53
OUTPUT	000590 - 000770	54
GRAPH	000810 - 001030	54

Subroutines:

BRODE	001060 - 001300	55
DTERM	001310 - 001610	55

##### 2. Program OUT-OF-PLANE

Main program:

<u>Section</u>	<u>Sequence number</u>	<u>Page</u>
INPUT	000100 - 000150	57
ITERATION AND EVALUATION OF ROOTS	000190 - 000590	57
OUTPUT	000630 - 000830	58
GRAPH	000870 - 001110	58



Subroutines:

BRODE	001140 - 001600	59
DTERM	001610 - 001910	60

B. LISTING







```

      GO TO 16
14  ROOT(N,J)=X(I)
15  WRITE(6,15) N, ROOT(N,J)
16  FORMAT(0,47X,I3,10X,F20.16,8HITER=300)
    IF(N.EQ.MODE) GO TO 500
20  N=N+1
    P(K+1)=P(K)+0.0500
110 CONTINUE
500 CGNTINUE

      C
      C
      C      OUTPUT
      C
1200 WRITE(6,1200)
      FORMAT(1,43X,12HCETIN DERIN,10X,14HDECEMBER 1970)
1300 WRITE(6,1300)
      FORMAT(0,116H*****
1 *****
      WRITE(6,1400)
      FORMAT(0,49X,19HIN PLANE VIBRATIONS)
1400 DO 30 L=1,NRO
      WRITE(6,1500) RO(L)
1500 FORMAT(0,53X,3HRO=,F7.5)
1600 WRITE(6,1600)
      FORMAT(0,50X,16H*****
      WRITE(6,1700)
1700 FORMAT(0,47X,4HMODE,16X,2HPL)
    DO 31 I=1,MODE
      WRITE(6,13) I,ROOT(I,L)
13  FORMAT(0,47X,I3,10X,F20.16)
31  CONTINUE
30  CGNTINUE

      C
      C
      C      GRAPH
      C
      IF(NGRAPH.EQ.0) GO TO 7000
      READ(5,8) (ITITLE(I),I=1,6)
      READ(5,8) (ITITLE(I),I=7,12)
8  FORMAT(6A8)
    DO 800 MM=1,MODE
      DO 700 M=1,NRO
        SEV(M)=ROOT(MM,M)
        CEM(M)=RO(M)
700 CONTINUE
      IF(MM.EQ.1) GO TO 4100
      IF(MM.GT.1.AND.MM.LT.MODE) GO TO 4200
      IF(MM.EQ.MODE) GO TO 4300
4100 CALL DRAW(NRO,CEM,SEV,1,0,LABEL,ITITLE,0.2,5.0,0.0,0.0,7,10,1,
      1LAST)

```

```

000470
000480
000490
000500
000510
000520
000530
000540
000550
000560
000570
000580
000590
000600
000610
000620
000630
000640
000650
000660
000670
000680
000690
000700
000710
000720
000730
000740
000750
000760
000770
000780
000790
000800
000810
000820
000830
000840
000850
000860
000870
000880
000890
000900
000910
000920
000930
000940

```





```

GO TO 150
4200 CALL DRAW(NRO,CEM,SEV,2,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
1 LAST)
GO TO 150
4300 CALL DRAW(NRO,CEM,SEV,3,0,LABEL,ITITLE,0.2,5.0,0,0,0,0,7,10,1,
1 LAST)
150 WRITE(6,4400) LAST
4400 FORMAT(IH0,5X,'LAST=',I2)
800 CONTINUE
7000 STOP
END

```

```

000950
000960
000970
000980
000990
001000
001010
001020
001030
001040
001050

```

```

SUBROUTINE BRODE(YY,RR,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(4,4)
Y=0.5D0
Z=0.0D0
U=YY*RR
A(1,1)=Y*(DCOSH(YY)-DCOS(YY))
A(1,2)=Z
A(1,3)=Y*(DSINH(YY)-DSIN(YY))
A(1,4)=Z
A(2,1)=Z
A(2,2)=Y*(DCOSH(U)-DCOS(U))
A(2,3)=Z
A(2,4)=Y*(DSINH(U)-DSIN(U))
A(3,1)=Y*(DSINH(YY)+DSIN(YY))
A(3,2)=-Y*(DSINH(U)+DSIN(U))
A(3,3)=A(1,1)
A(3,4)=-A(2,2)
A(4,1)=Y*(DCOSH(YY)+DCOS(YY))
A(4,2)=Y*(DCOSH(U)+DCOS(U))
A(4,3)=A(3,1)
A(4,4)=-A(3,2)
CALL DTERM(4,A,DET,4)
RETURN
END

```

```

001060
001070
001080
001090
001100
001110
001120
001130
001140
001150
001160
001170
001180
001190
001200
001210
001220
001230
001240
001250
001260
001270
001280
001290
001300

```

```

SUBROUTINE DTERM(N,A,D,M)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(M,M)
DC=1.D0
DO 34 L=1,N
KP=0
Z=0.0D0
DO 12 K=L,N

```

```

001310
001320
001330
001340
001350
001360
001370
001380

```



```

11 IF(Z-DABS(A(K,L)))11,12,12
12 Z=DABS(A(K,L))
13 KP=K
14 CONTINUE
15 IF(L-KP) 13,20,20
16 DO 14 J=L,N
17 Z=A(L,J)
18 A(L,J)=A(KP,J)
19 A(KP,J)=Z
20 DD=-DD
21 IF(L-N) 31,40,40
22 LP1=L+1
23 DO 34 K=LP1,N
24 IF(A(K,L)) 32,34,32
25 RATIO=A(K,L)/A(L,L)
26 DO 33 J=LP1,N
27 A(K,J)=A(K,J)-RATIO*A(L,J)
28 CONTINUE
29 DO 41 K=1,N
30 DD=DD*A(K,K)
31 D=DD
32 RETURN
33 END

```

```

001390
001400
001410
001420
001430
001440
001450
001460
001470
001480
001490
001500
001510
001520
001530
001540
001550
001560
001570
001580
001590
001600
001610

```



```

PROGRAM OUT-OF-PLANE
  IMPLICIT REAL*8(A-H,O-Z)
  REAL*4 SEV,CEM
  REAL*8 ITITLE(I2)
  REAL*4 LABEL/4H
  DIMENSION P(900),X(350),RO(60),ROOT(15,60,5),BETA(5),SEV(60),CEM(
160),F(900)
  INPUT
  EPS=1.0D-6
  READ(5,1) NRO,NBETA,MODE,NGRAPH
1  FORMAT(4I3)
  READ(5,2) (BETA(II),II=1,NBETA)
2  READ(5,2) (RO(JJ),JJ=1,NRO)
  FORMAT(7F10.0)
  ITERATION AND EVALUATION OF ROOTS
  DO 19 J=1,NRO
  DO 18 L=1,NBETA
  N=1
  P(1)=0.05D0
  CALL BRODE(P(1),RO(J),BETA(L),DET)
  F(1)=DET
  P(2)=0.1D0
  DO 17 K=2,850
  CALL BRODE(P(K),RO(J),BETA(L),DET)
  F(K)=DET
  B=F(K)*F(K-1)
  IF(B.GT.0.0D0) GO TO 20
  I=1
  XX1=P(K-1)
  XX2=P(K)
9  X(I)=(XX1+XX2)/2.0D0
  BBB=DABS(XX2-XX1)
  IF(BBB.LE.EPS) GO TO 12
  CALL BRODE(XX1,RO(J),BETA(L),DET)
  R3=DET
  CALL BRODE(X(I),RO(J),BETA(L),DET)
  R4=DET
  IF(R3.GT.0.0D0.AND.R4.GT.0.0D0) GO TO 10
  IF(R3.LT.0.0D0.AND.R4.LT.0.0D0) GO TO 10
  XX2=X(I)
  GO TO 11
10 XX1=X(I)
11 IF(I.EQ.300) GO TO 14

```









```

3100 CEM(MMM)=RO(MMM)
      CONTINUE
      IF(M.EQ.1.AND.MM.EQ.1) GO TO 4100
      IF(M.NE.MODE) GO TO 4200
      IF(M.EQ.MODE.AND.MM.EQ.NBETA) GO TO 4300
4100 CALL DRAW(NRO,CEM,SEV,1,0,LABEL,ITITLE,0.2,5.0,0,0,0,7,10,1,
      1, LAST)
      GO TO 4400
4200 CALL DRAW(NRO,CEM,SEV,2,0,LABEL,ITITLE,0.2,5.0,0,0,0,7,10,1,
      1, LAST)
      GO TO 4400
4300 CALL DRAW(NRO,CEM,SEV,3,0,LABEL,ITITLE,0.2,5.0,0,0,0,7,10,1,
      1, LAST)
4400 WRITE(6,4500) LAST
4500 FORMAT(1H0,5X,'LAST=',I2)
3200 CONTINUE
3300 CONTINUE
7000 STOP
      END
000950
000960
000970
000980
000990
001000
001010
001020
001030
001040
001050
001060
001070
001080
001090
001100
001110
001120
001130

```

```

SUBROUTINE BRODE(YY,RR,BB,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
Y=0.5D0
Z=0.0D0
U=YY*RR
W=BB*YY**2
V=W*RR
A(1,1)=Y*(DCOSH(YY)-DCOS(YY))
A(1,2)=-Y*(DCOSH(U)-DCOS(U))
A(1,3)=Z
A(1,4)=Z
A(1,5)=Y*(DSINH(YY)-DSIN(YY))
A(1,6)=-Y*(DSINH(U)-DSIN(U))
A(2,1)=Z
A(2,2)=Y*(DSINH(U)+DSIN(U))
A(2,3)=-DSIN(W)*1.3D0/BB
A(2,4)=Z
A(2,5)=Z
A(2,6)=-A(1,2)
A(3,1)=Y*(DSINH(YY)+DSIN(YY))
A(3,2)=Z
A(3,3)=Z
A(3,4)=DSIN(V)*1.3D0/BB
A(3,5)=A(1,1)
A(3,6)=Z
A(4,1)=A(1,5)
001140
001150
001160
001170
001180
001190
001200
001210
001220
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001240
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001380
001390
001400

```



```

A(4,2)=-A(1,6)
A(4,3)=Z
A(4,4)=Z
A(4,5)=Y*(DCOSH(YY)+DCOS(YY))
A(4,6)=Y*(DCOSH(U)+DCOS(U))
A(5,1)=A(4,5)
A(5,2)=Z
A(5,3)=Z
A(5,4)=-YY*DCOS(V)
A(5,5)=A(3,1)
A(5,6)=Z
A(6,1)=Z
A(6,2)=A(4,6)
A(6,3)=YY*DCOS(W)
A(6,4)=Z
A(6,5)=Z
A(6,6)=A(2,2)
CALL DTERM(6,A,DET,6)
RETURN
END

```

```

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001580
001590
001600

```

```

SUBROUTINE DTERM(N,A,D,M)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(M,M)
DD=1.0D0
DO 34 L=1,N
  KP=0
  Z=0.0D0
  DO 12 K=L,N
    IF(Z-DABS(A(K,L)))11,12,12
    Z=DABS(A(K,L))
  11 KP=K
  12 CONTINUE
  IF(L-KP) 13,20,20
  13 DO 14 J=L,N
    Z=A(L,J)
    A(L,J)=A(KP,J)
    A(KP,J)=Z
  14 DD=-DD
  20 IF(L-N) 31,40,40
  31 LP1=L+1
  DO 34 K=LP1,N
    IF(A(K,L)) 32,34,32
  32 RATIO=A(K,L)/A(L,L)
  DO 33 J=LP1,N
    A(K,J)=A(K,J)-RATIO*A(L,J)
  33 CONTINUE
  34

```

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001680
001690
001700
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001740
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001770
001780
001790
001800
001810
001820
001830
001840
001850
001860

```



```
40 DO 41 K = 1, N  
41 DD=DD*A(K, K)  
D=DD  
RETURN  
END
```

```
001870  
001880  
001890  
001900  
001910
```



## APPENDIX E

### PROGRAM VIPIPE

#### A. GENERAL

As a part of the present thesis investigation, attention was given to a digital computer program called VIPIPE which was originally written by Fink [1] and later modified and augmented by Kim [2]. This program provides solutions for in-plane and out-of-plane vibration of a single plane piping configuration having quite general geometry (except, of course, for its limitation to a single plane). It was desired to convert this program from a FORTRAN 63 program for use with CDC-1604 computer formerly employed at the Naval Postgraduate School, to a FORTRAN IV program for the IBM 360/67 presently in use, in order to provide an independent means of checking the results obtained as indicated in the body of this thesis and, also, because it would be useful to have a program of such generality available. For details of the theory of VIPIPE, refer to the theses written by Fink and Kim.

Because of the limitations on word length in IBM 360/67, the converted program, which was called VIPIPE-1, has been written to employ double precision arithmetic. Certain improvements have been made in the capability to produce graphical output.





The effort to make this conversion has been found to be much greater than was anticipated. It seems that CDC-1604 computer is a more "forgiving" machine than is the IBM 360/67. In checking out the new program certain bugs came to light which were actually present in the earlier version but which apparently did not cause trouble. However, they represented some syntactical errors that resulted in failure on the IBM 360/67. In some cases it was clear how to modify the program so as to eliminate the errors but in other cases the difficulty was so deep in the logic that a correction could not be made without acquiring as deep a knowledge of the program as was possessed by original writers, Fink and Kim.

Thus it cannot be asserted that VIPIPE-1 has been successfully converted in every fine detail and it is recommended that additional efforts be made to assure the overall integrity of the program and to assess its limitations. It should be noted, however, that no gross errors have been found in any results obtained by use of the new program; the only question is that of the degree of accuracy that it is possible to obtain.

Comparisons of accuracy are difficult to make. In the first place, the word-length is different in the new program, and, other things being equal, one would expect the new program to provide better accuracy, and to maintain its integrity in the face of round-off error beyond the point where the earlier programs "bombed out." Thus, agreement



between Kim and Fink and disagreement with VIPIPE-1 does not mean that the former are "correct" and the latter is in error. Furthermore, Kim reports having discovered and corrected some bugs which he found in Fink's version. Also, there is some difficulty in understanding the precise details of some of the problems treated by Fink and Kim.

With this preface which admonishes the reader not to make superficial inferences from comparisons, in Section C some comparisons between results obtained by Kim and results obtained using VIPIPE are presented.

## B. DATA DECK ASSEMBLY

The order of data deck is

1. Graph title data cards.
2. Problem statement card.
3. Boundary condition data card(s).
4. Branch joining angle data card(s).
5. Component extensive properties data card(s).
6. Component intensive properties data card(s).

Field specification and format of graph title cards are given in Section E-1-c of Appendix B and information for the remaining cards are given in [1].

## C. SAMPLE PROBLEMS

Properties of piping systems analyzed in this section are given in [1]. Here, only numerical values of frequencies will be given.



## 1. System 1 (in-plane)

Mode No.	Comparison* Values	VIPIPE** Delta m. (D)	VIPIPE-1*** Delta m.
1	75.09994540	75.09994544	75.09994535
2	207.01589137	207.01589036	207.01589099
3	405.83391916	405.83393183	405.83391910
4	670.86408383	670.86310856	670.86408365
5	1002.15520225	1002.19978814	1002.15520252
6	1399.70436269		1399.70434917
7	1863.51172599		1863.51198134

\*Sources of comparison frequencies for system 1 and system 2 are given in Section D.

\*\*Frequencies are given in [1].

\*\*\* Frequencies are found by VIPIPE-1.

## 2. System 2 (in-plane)

Mode No.	Comparison Values	VIPIPE Delta m. (D)	VIPIPE-1 Delta m.
1	11.80213586	11.80213389	11.80213432
2	73.96272297	73.96272167	73.96272087
3	207.09776595	207.09776685	207.09776559
4	405.82896584	405.82894132	405.82896568
5	670.86435913	670.86200817	670.86435889
6	1002.15518770	1002.26833752	1002.15518758
7	1399.70436229	1399.08849534	1399.70437163



### 3. System 4 (in-plane)

Mode No.	VIPIPE M-D m. (D)	VIPIPE-1 M-D m.
1	66.69934902	67.31642169
2	171.22352538	172.86857155
3	335.35007083	339.09491357
4	535.64529414	544.08929568
5	773.44358487	794.57343086

### 4. System 5 (in-plane)

Mode No.	VIPIPE Delta m. (D)	VIPIPE-1 Delta m.
1	116.89479180	118.95185774
2	304.88354398	302.91288556
3	566.43670976	519.19656352

Mode No.	VIPIPE P.M. m.-C (D)	VIPIPE-1 P.M. m. -C
1		118.95185779
2		302.91288592
3		519.19656389
4	826.88281929	794.49003694
5	1126.83797508	1126.27858174
6	1666.58878192	1464.59530724
7	2099.52844030	1874.97923215





#### D. SOURCES OF COMPARISON FREQUENCIES FOR SYSTEM 1 AND SYSTEM 2

The governing fourth order equation of a homogeneous single component straight system

$$y^{IV} = \frac{\mu\omega^4}{EI} y \quad (E-1)$$

was solved to give

$$y = a\cos Px + b \cosh Px + c \sin Px + d \sinh Px$$

Upon applying boundary conditions for system 1\* ( $y = y' = 0$  at  $x = 0, x = L$ ) and simplifying, it was found that the eigenvalues of the system must satisfy

$$\cos(PL) \cosh(PL) = 1 \quad (E-2)$$

Upon applying the boundary conditions for system 2 ( $y = y' = 0$  at  $x = 0, y'' = y''' = 0$  at  $x = L$ ), it was found that the eigenvalues of the system must satisfy

$$\cos(PL) \cosh(PL) = -1 \quad (E-3)$$

The roots of equation E-2 and E-3, from which the natural frequencies of corresponding systems may easily be found, were obtained through iteration using a digital computer.

---

\*System 1 is clamped-clamped end beam  
System 2 is clamped-free end beam



## E. PROGRAM LISTING

### 1. Index

#### Main program:

<u>Section</u>	<u>Sequence number</u>	<u>Page</u>
INPUT	000010 - 001080	70
INVARIANTS	001120 - 001520	72
CONTROL IN PLANE	001560 - 002470	73
CONTROL OUT OF PLANE	002520 - 003420	75
ITERATION	003460 - 004460	77
OUTPUT	004490 - 008135	79

#### Subroutines:

SUBSEC	008140 - 008640	87
DISTM	008670 - 009220	88
DISTMO	009652 - 009676	89
RIGID	009652 - 009676	91
RIGIO	009679 - 009730	91
STIFCO	009760 - 009970	92
STIFOO	010000 - 010230	92
STAVEC	010260 - 010550	93
STAVEO	010580 - 010830	94
INVERT	010860 - 011570	94
HANGER	011600 - 011770	96
HANGE0	011800 - 011970	96
CFIELO	012000 - 012520	97
POINO	012550 - 012770	98
CFIELD	012000 - 012520	98
POINT	012550 - 013490	99



SFIELD	013520 - 013880	100
SFIELO	013910 - 014180	101
MATMUL	014210 - 014330	101
FINBRA	014360 - 014620	102
FINMAT	014650 - 014920	102
BRANCH	014950 - 015580	103
BRANCO	015610 - 016290	104
STATEM	016320 - 016840	106
STATEB	016870 - 017610	107
BRCOR	017630 - 017940	108
DELMA	017970 - 019380	109

## 2. Listing



PROGRAM VIPIPE-1

INPUT

IMPLICIT REAL\*8(A-H,O-Z)

REAL\*4 CET,ZERGU

REAL\*8 ITITLE(12)

REAL LABEL//ONE,/,

REAL LABEL1//TWO,/,

REAL LABEL2//THRE,/,

REAL LABEL3//FOUR,/,

REAL LABEL4//FIVE,/,

REAL LABEL5//SIX,/,

REAL LABEL6//SEVN,/,

REAL LABEL7//EIT,/,

REAL LABEL8//NINE,/,

REAL LABEL9//TEN,/,

REAL LABEL10//LAST,/,

COMMON AJT(50),R(50),AIX(50),AIY(50),PHI(20),SVI(6,1),

1DT(50),D(50),DI(50),TL(50),Z(50),G(50),E(50),AMU(50),THETA(50),RHO

2(50),OMEGAM(20),DEL(600),OMEGA(600),A(6,6),B(6,6),U(6,6),CLX(50),C

3LY(50),CLZ(50),CTX(50),CTY(50),CTZ(50),V(6,3),VV(6,3),UU(6,3),SVE(

46,1),SV(22),SVIP(6,22),SVO(22),SVOP(6,22),NNBR(50),BAMU(50),THETR(

520),P(20),YY(3,3,23),R3(3,3),XX(3,3),EDL(600),ZERGU(600),CET(600)

SEVIN=1.0D0

EEVIN=0.0D0

NM=0

JJ=1

READ(5,2001) (ITITLE(I),I=1,6)

READ(5,2001) (ITITLE(I),I=7,12)

FORMAT(6A8)

2001 READ(5,1500) NS,NBR,IOP,HM,AMYK,SD,RI,OMEGAG,OMEGAI,GDABC,BRC,

450 GRAPH,REM,PNM,HL

1500 FORMAT(3I2,4F3.0,2F8.0,4F2.0,2F4.0)

IF(NS) 600,600,601

601 NMBR=NBR+2

IF(IOP-1) 1501,1501,1506

1501 READ(5,1502) (SV(I),I=1,NMBR)

1502 FORMAT(2F3.0)

DO 1505 J=1,NMBR

IF(SV(J)) 1503,1503,1505

1503 READ(5,1504) (SVIP(I,J),I=1,6)

1504 FORMAT(6F2.0)

1505 CONTINUE

IF(IOP) 1506,1506,1523

1506 READ(5,1507) (SVO(I),I=1,NMBR)

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000240  
000250  
000260  
000270  
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000290  
000300  
000310  
000320





```

1507 FORMAT(22F3.0)
DO 1510 J=1,NMBR
IF(SVC(J)) 1508,1510
1508 READ(5,1509) (SVOP(I,J),I=1,6)
1509 FORMAT(6F2.0)
1510 CONTINUE
1523 IF(NBR) 1513,1513,1511
1511 READ(5,1512) (PHI(I),I=1,NBR)
1512 FORMAT(10F7.3)
1513 READ(5,1514) (D(I),DI(I),RHO(I),TL(I),THETA(I),CLX(I),CLY(I),CLZ
1(I),CTX(I),CTY(I),CTZ(I),NNBR(I),I=1,NS)
1514 FORMAT(3F5.2,2F6.2,6F7.2,I2)
1515 READ(5,1515) AAMU,AE,AG,DD
1515 FORMAT(F7.3,2E10.2,F2.0)
1516 IF(DD) 1516,1516,1518
DO 1517 I=1,NS
AMU(I)=AAMU
E(I)=AE
G(I)=AG
1517 GO TO 1520
1518 AMU(1)=AAMU
E(1)=AE
G(1)=AG
1519 READ(5,1519) (AMU(I),E(I),G(I),I=2,NS)
1519 FORMAT(F7.3,2E10.2)
1520 IF(OMEGAI) 1121,1121,1521
1121 OMEGAI=0.01D0
1521 OMEGA(1)=OMEGAI
ANYK=AMVK-SEVIM
OMEGAO=OMEGAI
FFAC=0.5D0
L=1
M=1
MN=0
AA=EEVIN
BB=EEVIN
CC=EEVIN
FF=EEVIN
GG=EEVIN
RR=EEVIN
KK=0
PP=EEVIN
KA=0
KKK=0
AAMU=EEVIN
EE=EEVIN
AL=EEVIN
AD=EEVIN

```



```

NNN=NBRR+1
IF(PMM*0.1D0-SEVIM) 6091,6092,6093
6091 PM=-SEVIM
GO TO 7000
6092 PM=EEVIN
GO TO 7000
6093 IF(PMM*0.01D0-SEVIM) 6094,6095,6096
6094 PM=-SEVIM
GO TO 7000
6095 PM=SEVIM
GO TO 7000
6096 IF(PMM*0.01D0-1.1D0) 6097,6098,6099
6097 PM=-SEVIM
GO TO 7000
6098 PM=EEVIN
GO TO 7000
6099 PM=-SEVIM
7000 PPM=PM
IF(PM) 7073,7073,7001
7001 IF(REM-SEVIM) 7071,7072,7071
7071 SS=-SEVIM
GO TO 7073
7072 SS=EEVIN
7073 DO 1522 I=1,NS
DI(I)=DI(I)
BAMU(I)=AMU(I)
THEIR(I)=THETA(I)
1522 DI(I)=D(I)-2.0D0*DI(I)
C
C
C
C
INVARIENTS
DO 108 I=1,NS
IF(CLY(I)+CLY(I)+CLZ(I)+CTX(I)+CTY(I)+CTZ(I)) 108,101,108
101 AMU(I)=AMU(I)*3.141592654D0*(D(I)**2-DI(I)**2)/(6912.0D0*32.17D0
1*12.0D0)
IF(RHO(I)) 102,103,102
102 THETA(I)=THETA(I)*3.141592654D0/180.0D0
TL(I)=THETA(I)*DABS(RHO(I))
103 AJT(I)=3.141592654D0*(D(I)**4-DI(I)**4)/32.0D0
AJ(I)=AJT(I)/2.0D0
R(I)=SEVIM/(AJ(I)*E(I))
AIX(I)=DSQRT((D(I)**2+DI(I)**2)/8.0D0)
AIF(AMK) 104,105,104
104 Z(I)=SEVIM
IF(RHO(I)) 105,106,105
105 HMM=HM

```



```

HMH=EEVIN
CALL SUBSEC (RHO(I),THETA(I),D(I),TL(I),HM,Z(I),OMEGA I,HMH)
HM=HMH
106 IF(NNBR(I)) 108,107,108
107 AL=AL+TL(I)
AD=AD+D(I)*TL(I)
EE=EE+E(I)*TL(I)
AID=AID+DI(I)*TL(I)
AAMU=AAMU+AMU(I)*TL(I)
108 CONTINUE
AD=AD/AL
AID=AID/AL
AFR=3.141592654D0/64.0D0*(AD**4-AID**4)
FR=4.73004019D0**2*DSQRT(EE*AFR/(AAMU*AL**4))
ADOM=FR/4.0D0
DCM=ADOM
CR=ADOM*0.001D0
BDOM=ADOM
BR=CR
ERM=REM
DO 109 I=1,NBR
109 PHI(I)=PHI(I)*3.141592654D0/180.0D0
IF(IOP-1) 110,999,998
110 MN=1
CC
CC
CC
CONTROL IN PLANE
999 P1=SEVIM
P2=SEVIM
D1=EEVIN
D2=EEVIN
Q1=SEVIM
Q2=SEVIM
Q3=SEVIM
B1=EEVIN
B2=EEVIN
B3=EEVIN
FINKEEVIN
IF(SV(1)) 1011,1021,1011
CALL STAVEC (SV(1),1,SVI)
1011 GO TO 1041
DO 1031 J=1,6
1031 SVI(J,1)=SVIP(J,1)
1041 IF(SV(NNBR)) 1051,1061,1051
1051 CALL STAVEC(SV(NNBR),1,SVE)
GO TO 1081
1061 DO 1071 J=1,6

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1071 SVE(J,1)=SVIP(J,NMBR)
1081 JK=0
1085 IF(NBR) 1000,1000,1085
1085 DO 1087 N=2,NNN
1086 IF(SV(N)) 1087,1087,1086
1087 CALL STAVEC(SV(N),N,SVIP)
1000 CONTINUE
N=2
II=1
HH=EEVIN
DO 1001 I=1,NS
MAT=0
BC=EEVIN
P(I)=CLX(I)+CLY(I)+CTZ(I)
IF(P(I)) 37,38,37
37 CALL HANGER(CLX(I),CLY(I),CTZ(I),U)
GO TO 62
38 IF(AMK) 39,41,39
39 IF(RHO(I)) 40,42,40
40 IF(Z(I)) 43,45,43
41 IF(RHO(I)) 40,48,40
42 IF(Z(I)) 46,47,46
43 CALL CFIELD(R(I),Z(I),G(I),SD,RHO(I),THETA(I),D(I),DI(I),E(I),A)
44 CALL POINT(AMU(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)
CALL MATMUL(A,B,U)
MAT=1
GO TO 62
45 CALL STIFCO(THETA(I),RHO(I),U)
GO TO 62
46 CALL DISTM(AMU(I),AIY(I),OMEGA(L),TL(I),R(I),D(I),DI(I),G(I),SD,
1RI,E(I),FFAC,U)
GO TO 62
47 CALL RIGID(AMU(I),TL(I),OMEGA(L),AIY(I),RI,U)
GO TO 62
48 IF(Z(I)) 49,47,49
49 CALL SFIELD(DI(I),TL(I),R(I),Z(I),G(I),SD,D(I),E(I),FFAC,A)
GO TO 44
62 IF(NBR(I)) 81,80,81
80 IF(I-1) 63,79,63
79 CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
GO TO 63
63 CALL FINMAT(U,UU,V,MAT,A,Z(I))
DO 64 J=1,6
DO 64 K=1,3
UU(J,K)=V(J,K)
64 IF(PM) 88,113,84
1113 IF(FINK) 88,84,88
84 IF(BC) 88,88,1111

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002240

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1111	IF(BRC) 88,88,89	002250
89	CALL BRCOR(R3,UU,XX,JK,VV,PM)	002260
	MMM=N-1	002270
	DO 90 J=1,3	002280
	DO 90 K=1,3	002290
90	YY(J,K,MMM)=XX(J,K)	002300
88	GO TO 1001	002310
81	IF(NNBR(I)-1) 83,83,65	002320
83	CALL STATEB(SVIP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)	002330
65	CALL FINBRA(U,VV,V,MAT,A,Z{I})	002340
	DO 66 J=1,6	002350
	DO 66 K=1,3	002360
66	VV(J,K)=V(J,K)	002370
	IF(NNBR(I)) 68,67,67	002380
67	IF(NNBR(I)-3) 1001,68,68	002390
68	CALL BRANCH(R3,VV,PHI(II),U)	002400
	N=N+1	002410
	II=II+1	002420
	MAT=0	002430
	BC=SEVIM	002440
	GO TO 63	002450
	CONTINUE	002460
1001	GO TO 2000	002470
		002480
		002490
		002500
		002510
		002520
		002530
		002540
		002550
		002560
		002570
		002580
		002590
		002600
		002610
		002620
		002630
		002640
		002650
		002660
		002670
		002680
		002690
		002700
		002710
		002720

1013	IF(SVO(I)) 1013,1023,1013
	CALL STAVEO(SVO(I),I,SVI)
1013	GO TO 1043
1023	DO 1033 J=1,6
1033	SVI(J,I)=SVOP(J,I)
1043	IF(SVO(NNBR)) 1053,1063,1053
1053	CALL STAVEO(SVO(NNBR),I,SVE)
	GO TO 1083
1063	DO 1073 J=1,6
1073	SVE(J,I)=SVOP(J,NMBK)

998	P1=SEVIM
	P2=SEVIM
	D1=EEVIN
	D2=EEVIN
	Q1=SEVIM
	Q2=SEVIM
	Q3=SEVIM
	B1=EEVIN
	B2=EEVIN
	B3=EEVIN
	FINK=EEVIN

	CONTROL OUT OF PLANE
--	----------------------

C	
C	
C	



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1083 JK=1
1093 IF(NBR) 1002,1002,1093
1095 DO 1095 N=2,NNN
1094 IF(SVD(N)) 1095,1095,1094
1095 CALL STAVEO(SVD(N),N,SVOP)
1002 CONTINUE
N=2
II=1
HH=EEVIN
DO 1003 I=1,NS
BC=EEVIN
MAT=0
P(I)=CTX(I)+CLZ(I)+CTY(I)
IF(P(I)) 1,2,1
1 CALL HANGEQ(CTX(I),CLZ(I),CTY(I),U)
GO TO 14
2 IF(AMK) 3,11,3
3 IF(RHO(I)) 4,8,4
4 IF(Z(I)) 6,5,6
5 CALL STIFQO(THETA(I),RHO(I),U)
GO TO 14
6 CALL CFIELO(AJT(I),R(I),Z(I),G(I),SD,RHO(I),THETA(I),DI(I),FF
IAC,A)
7 CALL POINO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),Z(I),RI,B)
CALL MATMUL(A,B,U)
MAT=1
GO TO 14
8 IF(Z(I)) 10, 9,10
9 CALL RIGIO(AMU(I),TL(I),AIX(I),OMEGA(L),AIY(I),AJT(I),RI,U)
GO TO 14
10 CALL DISTMO(AMU(I),AIX(I),AIY(I),OMEGA(L),TL(I),AJT(I),R(I),D(I),
DI(I),G(I),SD,RI,FFAC,U)
GO TO 14
11 IF(RHO(I)) 4,12,4
12 IF(Z(I)) 13,9,13
13 CALL SFIELO(DI(I),TL(I),AJT(I),R(I),Z(I),G(I),SD,D(I),FFAC,A)
GO TO 7
14 IF(NBR(I)) 201,200,201
200 IF(I-1) 15,202,15
202 CALL STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
GO TO 15
15 CALL FINMAT(U,UU,V,MAT,A,Z(I))
DO 16 J=1,6
DO 16 K=1,3
16 UU(J,K)=V(J,K)
IF(PM) 207,1114,85
1114 IF(FINK) 207,85,207
85 IF(BC) 207,207,1112

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1112 IF(BRC) 207,207,208
208 CALL BRCOR(R3,UU,XX,JK,VV,PM)
      MMM=N-1
      DO 209 J=1,3
      DO 209 K=1,3
      YY(J,K,MMM)=XX(J,K)
207 GO TO 1003
201 IF(NNBR(I)-1) 204,204,17
204 CALL STATEB(SVOP,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
17 CALL FINBRA(U,VV,V,MAT,A,Z(I))
      DO 18 J=1,6
      DO 18 K=1,3
      VV(J,K)=V(J,K)
18 IF(NNBR(I)) 82,19,19
19 IF(NNBR(I)-3) 1003,82,82
82 CALL BRANCO(R3,VV,PHI(II),U)
      N=N+1
      II=II+1
      MAT=0
      BC=SEVIM
      GO TO 15
      CONTINUE

1003
C
C
C
2000 CALL DELMA(UU,SVE,V,X2,Y2,KKK,X1,FF,PP,HH,RR,SS,Y1,REM,FINK,PM,D1,
102)
      IF(FINK) 87,87,86
87 IF(PM) 99,99,98
98 IF(KKK) 7030,7030,301
301 GO TO 6001
7030 IF(FF-SEVIM) 7051,7060,7051
7060 KK=1
      M=M-1
      GO TO 3000
7051 IF(HH-SEVIM) 300,6001,300
300 IF(SS) 303,302,302
303 DEL(L)=X2-Y2
      D1=X1
      D2=(X2+Y2)/2.0D0
      GO TO 61
302 DEL(L)=X1-Y1
      D1=(X1+Y1)/2.0D0
      D2=X2
      GO TO 61
99 DEL(L)=V(1,1)*V(2,2)*V(3,3)-V(1,3)*V(2,2)*V(3,1)+V(1,2)*V(2,3)*
1V(3,1)-V(1,2)*V(2,1)*V(3,3)+V(1,3)*V(2,1)*V(3,2)*
2V(2,3)

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003210
003220
003230
003240
003250
003260
003270
003280
003290
003300
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003505
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003670

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003970
003980
003990
004000
004010
004020
004030
004040
004060
004070
004080
004090
004100
004110
004120
004130
004140
004150
004160

GO TO 61
21 PDEL=DEL(L)
GO TO 25
22 IF(AA) 23,23,24
23 PDEL=DEL(L-1)
GO TO 25
24 PDEL=DELC
25 IF(PDEL*DEL(L)) 26,26,27
26 IF(AA) 28,28,57
27 BB=EEVIN
28 AA=SEVIM
BB=SEVIM
DELC=DEL(L-1)
OMEGAC=OMEGA(L-1)
GO TO 59
29 OMEGAM(M)=OMEGAC+(OMEGA(L)-OMEGAC)*DABS(DELC)/(DABS(DELC))+
1DABS(DEL(L))
GO TO 32
30 OMEGA(L+1)=OMEGA(L)-0.9D0*DOM
DOM=0.5D0*DOM
31 L=L+1
BL=L
IF(HL-BL) 7053,7053,86
7053 KA=1
GO TO 3000
86 IF(JK) 1000,1000,1002
32 BM=M
IF(HM-BM) 3000,3000,33
33 M=M+1
IF(M-3) 340,34,34
34 ADOM=(OMEGAM(M-1)-OMEGAM(M-2))*0.1D0
340 AA=EEVIN
BB=EEVIN
CC=EEVIN
GG=EEVIN
DOM=ADOM
DELC=EEVIN
OMEGA(L+1)=OMEGAM(M-1)+2.0D0*CR
OMEGA(L+1)=OMEGA(L+1)
OMEGA(L+1)=OMEGA(L+1)
GO TO 31
35 IF(OMEGA(L)-OMEGAC) 53,60,60
50 OMEGA(L+1)=OMEGA(L)-(OMEGA(L)-OMEGAC)/2.0D0
51 IF(OMEGA(L)-OMEGA(L+1)-CR) 51,51,31
51 CC=SEVIM
GO TO 31
52 DELC=DEL(L)
OMEGAC=OMEGA(L)

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53 OMEGA(L+1)=OMEGA(L)+DOM
54 GO TO 31
55 IF(DEL C*DEL(L)) 29,29,55
56 OMEGA(M)=OMEGA(L)+(OMEGA(L-1)-OMEGA(L))*DABS(DEL(L))/(DABS(DEL(L)
1)+DABS(DEL(L-1)))
57 GO TO 32
58 IF(OMEGA(L)-OMEGAO) 21,21,22
59 IF(BB) 58,58,50
60 BB=SEVIM
59 IF(OMEGA(L)-OMEGA(L-1)-CR) 29,29,30
60 M=M-1
61 GO TO 3000
62 IF(GG) 70,70,72
70 IF(DEL(L)) 77,71,77
71 GG=SEVIM
72 OMEGA(L+1)=OMEGA(L)-CR
73 GO TO 31
74 IF(GG-SEVIM) 73,73,74
75 GG=2.000
76 HDEL=DEL(L)
77 OMEGA(L+1)=OMEGA(L)+2.000*CR
78 GO TO 31
79 TDEL=DEL(L)*HDEL
80 IF(TDEL) 75,76,76
81 OMEGA(M)=OMEGA(L-2)
82 GO TO 33
83 KK=1
84 M=M-1
85 GO TO 3000
86 IF(CC) 56,56,54
87 OUTPUT
88
89 3000 IF(NM) 400,400,403
90 400 WRITE(6,401)
91 401 FORMAT(1H1//)
92 402 WRITE(6,402)
93 402 FORMAT(16HPROGRAM VIPIPE-1,34X,15H M.CETIN DERIN,38X,14HDECEMBER
1 1970//,10X,108H
2 1 SYSTEM ARE DETERMINED BY AN ITERATIVE PROCEDURE USING THE /29H ME
3 THOD OF TRANSFER MATRICES.//119H *****
4 *****
5 *****
603 IF(NM)4058,4058,404
604 WRITE(6,405)
605 FORMAT(1H1//)
606 NM=1
607 4058 IF(JK) 4060,4060,4065
608 4059 IF(JK) 4060,4060,4065

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4060 WRITE(6,4070) JJ                                004650
4070 FORMAT(50X,8HPROBLEM I2,/,35X,47HIN PLANE MODE FREQUENCIES (RADIA 004660
      1NS PER SECOND) //45X,4HMODE,15X,10H FREQUENCY /) 004670
      GO TO 4079 004680
4065 WRITE(6,4075) JJ                                004690
4075 FORMAT(50X,8HPROBLEM I2,/,33X,51HOUT OF PLANE MODE FREQUENCIES (RA 004700
      1DIANS PER SECOND) //45X,4HMODE,15X,10H FREQUENCY /) 004710
      MH=M 004720
4079 WRITE(6,408) (M,OMEGAM(M),M=1,MH) 004730
      FORMAT(46X,I2,17X,F25.16 /) 004740
      IF(KA) 5000,5000,5010 004750
5010 WRITE(6,5020) 004760
5020 FORMAT(10X,53HPROBLEM TERMINATED DUE TO OMEGA STORAGE LIMITATION. 004770
      1 /) 004780
5000 IF(KK) 4080,4090,4080 004790
4080 WRITE(6,4085) 004800
4085 FORMAT(10X,59HPROBLEM TERMINATED DUE TO SIGNIFICANT FIGURE LIMITAT 004810
      1ION OF COMPUTER. /) 004820
4090 IF(AMK) 412,409,412 004830
409 WRITE(6,411) 004840
411 FORMAT(16X,53H 004850
      1ND ) A LUMPED MASS APPROACH WAS EMPLOYED, A
      GO TO 414 004860
412 WRITE(6,413) 004870
413 FORMAT(24X,45HA DISTRIBUTED MASS APPROACH WAS EMPLOYED AND ) 004880
414 IF(SD) 415,415,420 004890
415 IF(RI) 416,416,418 004900
416 WRITE(6,417) 004910
417 FORMAT(2X,67HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA W 004920
      1ERE CONSIDERED. /) 004930
      GO TO 4610 004940
418 WRITE(6,419) 004950
419 FORMAT(3X,63HTHE EFFECTS OF SHEAR DEFLECTION WERE CONSIDERED WHIL 004960
      1E THOSE OF /3X,35HROTATIONAL INERTIA WERE NEGLECTED. /) 004970
      GO TO 4610 004980
420 IF(RI) 421,421,423 004990
421 WRITE(6,422) 005000
422 FORMAT(3X,72HTHE EFFECTS OF ROTATIONAL INERTIA WERE CONSIDERED WH 005010
      1ILE THOSE OF SHEAR /3X,27HDEFLECTION WERE NEGLECTED. /) 005020
      GO TO 4610 005030
423 WRITE(6,424) 005040
424 FORMAT(3X,66HTHE EFFECTS OF SHEAR DEFLECTION AND ROTARY INERTIA WE 005050
      1RE NEGLECTED. /) 005060
4610 IF(PM) 425,425,4713 005070
4713 IF(SS) 4611,4612,4612 005080
4611 IF(PP-SEVIM) 4613,4614,4615 005090
4613 WRITE(6,4623) 005100
      FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(A) USED. /) 005110
      005120

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4614 GO TO 425
4624 WRITE(6,4624)
      FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(B) USED. /)
4615 GO TO 425
4625 WRITE(6,4625)
      FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(C) USED. /)
4612 GO TO 425
4616 IF(SS-SEVIM) 4616,4617,4618
4616 WRITE(6,4626)
4626 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(D) USED. /)
      GO TO 425
4617 WRITE(6,4627)
4627 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(E) USED. /)
      GO TO 425
4618 WRITE(6,4628)
4628 FORMAT(24X,35HPESTEL MAHRENHOLTZ METHOD(F) USED. /)
425 IF(PM)9010,9020,9000
9010 WRITE(6,9012)
9012 FORMAT(24X,20HDELTA METHOD USED. /)
      GO TO 9000
9020 WRITE(6,9022)
9022 FORMAT(24X,29HMODIFIED DELTA METHOD USED. /)
9000 WRITE(6,440)
440 *****
      FORMAT(118H *****
1 *****
      IF(GDABC-SEVIM)4511,4520,4401
4401 WRITE(6,4402)
4402 FORMAT(35H SECTION LENGTH AND SUBSECTION DATA/23X,14HSECTION NUMBE
      R,8X,25HLENGTH OF SECTION(INCHES),5X,21HNUMBER OF SECTIONS
      WRITE(6,443) (I,IL(I),Z(I),I=1,NS)
443 FORMAT (29X,13,23X,F7.2,21X,F4.0)
      LM=L
7055 IF(PM) 7054,7054,7055
441 WRITE(6,441)
      FORMAT (11H GRAPH DATA /22X,16HITERATION NUMBER,4X,30HFREQUENCY (R
      1ADIANS PER SECOND),3X,30H VALUE OF REMAINDER
      GO TO 7057
7054 WRITE(6,7056)
7056 FORMAT (11H GRAPH DATA / 22X,16HITERATION NUMBER,4X,30HFREQUENCY (R
      1ADIANS PER SECOND),3X,30HVALUE OF FREQUENCY DETERMINANT
7057 WRITE(6,444) (L,OMEGA(L),DEL(L),L=1,LM)
444 FORMAT (29X,13,23X,F11.6,18X,D15.8)
4520 WRITE(6,442)
442 FORMAT(1X,10HINPUT DATA /)
      WRITE(6,4521)
4521 FORMAT(1X,9HCOMPONENT,4X,8HDIAMETER,5X,14HWALL THICKNESS,5X,6HLEN
      1GTH,5X,9HRADIUS OF,9X,8HINCLUDED,5X,7HDENSITY,5X,7HELASTIC,5X,5HSH
      2EAR /57X,9HCURVATURE,5X,12HANGLE OF ARC,16X,7HMODULUS,5X,7HMODULUS

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005130
005140
005150
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005210
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005240
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005280
005290
005300
005310
005320
005330
005340
005350
005360
005370
005380
005390
005400
005410
005420
005430
005440
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005470
005480
005490
005500
005510
005520
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005570
005580
005590
005600

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3 / )
DO 4525 I=1,NS
IF(P(I))4524,4524,4523
4524 WRITE(6,4522) I,D(I),DT(I),TL(I),RHO(I),THETR(I),BAMU(I),E(I),G(I)
4522 FORMAT (3X,I3,8X,F7.3,10X,F6.3,7X,F8.3,4X,F8.3,9X,F6.2,8X,F7.2,4X,
1E8.2,4X,E8.2)
GO TO 4525
4523 SH=SEVIM
4525 CONTINUE
IF(SH)395,395,4526
4526 WRITE(6,4532)
4532 FORMAT(/1X,7HHANGERS /)
4527 WRITE(6,4527)
4527 FORMAT (2X,9HCOMPONENT,14X,3HCLX,14X,3HCLY,14X,3HCLZ,14X,3HCTX,14X
1,3HCTY,14X,3HCTZ /)
4528 DO 4530 I=1,NS
IF(P(I))4530,4530,4529
4529 WRITE(6,4531) I,CLX(I),CLY(I),CLZ(I),CTX(I),CTY(I),CTZ(I)
4531 FORMAT(3X,I3,15X,F10.2,7X,F10.2,7X,F10.2,7X,F10.2,7X,F10.
12)
4530 CONTINUE
395 WRITE(6,448)
448 FORMAT (/20H BOUNDARY CONDITIONS )
449 WRITE(6,449)((SVI(I,1),I=1,6)
FORMAT (5H SVI=6F3.0)
451 WRITE(6,451) ((SVE(I,1),I=1,6)
FORMAT (5H SVE=6F3.0)
IF(NBR) 4511,4511,4502
4502 IF(JK)4505,4505,4515
4505 WRITE(6,4510)((SVIP(I,J),I=1,6),J=2,NNN)
4510 FORMAT (6F3.0)
GO TO 4511
4515 WRITE(6,4517)((SVOP(I,J),I=1,6),J=2,NNN)
4517 FORMAT(6F3.0)
4511 CONTINUE
C
SORT
IF(GRAPH-SEVIM) 6001,6000,6001
6000 LM1=LM-1
DO 6100 L=1,LM
EDL(L)=DEL(L)
CONTINUE
6100 DO 6600 L=1,LM1
LP1=L+1
DO 6600 M=LPI,LM
IF(OMEGA(L)-OMEGA(M)) 6600,6600,6003
6003 TEMP=OMEGA(L)
OMEGA(L)=OMEGA(M)
OMEGA(M)=TEMP

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005610
005620
005630
005640
005650
005660
005670
005680
005690
005700
005710
005720
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005770
005780
005790
005800
005910
005920
005930
005940
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005970
005980
005990
006000
006010
006020
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006060
006070
006080
006090
006100
006110
006120
006130
006140
006150
006160
006170
006180

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TEMP=EDL(L)	006190
EDL(L)=EDL(M)	006200
EDL(M)=TEMP	006210
CONTINUE	006220
GRAPH	006230
NUMPTS=60	006240
LT1=LM/NUMPTS	006250
IF(LT1) 6024,6025,6024	006255
NUMPTS=LM	006270
L=1	006285
DO 2015 J=1,60	006300
ZERGU(J)=OMEGA(J)	006315
CET(J)=EDL(J)	006330
CONTINUE	006345
CALL DRAW(NUMPTS,ZERGU,CET,0,0,LABEL,ITITLE,0,0,0,0,0,9,10,1,	006360
1 LAST)	006375
GO TO 6001	006390
6024 DO 6020 J=1,17	006400
LT2=0	006410
LT2=LT2+J*NUMPTS	006420
IF(LT2-LM)6020,6021,6022	006430
CONTINUE	006440
LT=LT1	006450
GO TO 6023	006460
LT=LT1+1	006470
NUMEND=LM-LT1*NUMPTS	006480
L=1	006490
DO 7021 I=1,LT	006500
NEVIN1=I-3	006505
NEVIN2=I-5	006510
NEVIN3=I-7	006515
NEVIN4=I-9	006520
NEVIN5=I-LT	006525
IF(I-1) 6008,6009,6008	006530
IF(I-LT) 8011,6011,6011	006535
IF(I-3) 6010,6010,8012	006540
IF(I-5) 6010,6010,8013	006545
IF(I-7) 6010,6010,8014	006550
IF(I-9) 6010,6010,8015	006555
IF(I-LT) 6010,6011,6011	006560
DO 6031 N=1,NUMPTS	006780
EDL(N)=EDL(L+N-1)	006790
CET(N)=EDL(N)	006795
OMEGA(N)=OMEGA(L+N-1)	006800
ZERGU(N)=OMEGA(N)	006805
WRITE(6,6027) NUMPTS,EDL(1)	006810
FORMAT(1H1,////5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)	006820
CALL DRAW(NUMPTS,ZERGU,CET,1,0,LABEL1,ITITLE,0,0,0,0,0,9,10,1,	006830



```

1 LAST)
GO TO 7021
6010 L=L+NUMPTS
OMEGAX=OMEGA(L)
DO 6030 N=1,NUMPTS
EDL(N)=EDL(N+L-1)
CET(N)=EDL(N)
OMEGA(N)=OMEGA(L+N-1)
OMEGA(N)=OMEGA(N)-OMEGAX
ZERGU(N)=OMEGA(N)
6030 WRITE(6,6032) NUMPTS,EDL(1)
6032 FORMAT(///5X,7HNUMPTS=,I3,5X,7HEDL(1)=,E12.9)
IF(NEVIN1.LT.0) GO TO 2002
IF(NEVIN1.EQ.0) GO TO 2003
IF(NEVIN2.LT.0) GO TO 2004
IF(NEVIN2.EQ.0) GO TO 2005
IF(NEVIN3.LT.0) GO TO 2006
IF(NEVIN3.EQ.0) GO TO 2007
IF(NEVIN4.LT.0) GO TO 2008
IF(NEVIN4.EQ.0) GO TO 2009
IF(NEVIN5.LT.0) GO TO 2010
2002 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL2,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2003 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL3,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2004 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL4,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2005 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL5,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2006 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL6,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2007 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL7,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2008 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL8,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2009 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL9,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021
2010 CALL DRAW(NUMPTS,ZERGU,CET,2,0,LABEL0,ITITLE,0,0,0,0,0,0,9,10,1,
1 LAST)
GO TO 7021

```



6011	L=L+NUMPTS	006970
	OMEGAX=OMEGA(L)	006980
	DO 6033 N=1,NUMEND	006990
	EDL(N)=EDL(N+L-1)	007000
	CET(N)=EDL(N)	007005
	OMEGA(N)=OMEGA(N+L-1)	007010
	OMEGA(N)=OMEGA(N)-OMEGAX	007020
6033	ZERGU(N)=OMEGA(N)	007025
	WRITE(6,6051) NUMEND,EDL(1)	007030
6051	FORMAT(/////5X,7HNUMEND=,I3,5X,7HEDL(1)=,E12.9)	007040
	CALL DRAW(NUMEND,ZERGU,CET,3,0,0,0,0,0,0,9,10,1,	007050
	LAST)	007060
	GO TO 6001	007080
7021	CONTINUE	007090
6001	IF(HH-SEVIM) 8024,8023,8023	007100
8023	MH=M	007110
	LM=L	
8024	DO 446 M=1,MH	007130
446	OMEGAM(M)=EEVIN	007140
	DO 447 L=1,LM	007150
	OMEGA(L)=EEVIN	007160
	EDL(L)=EEVIN	007170
447	DEL(L)=EEVIN	007180
	DELC=EEVIN	007190
	OMEGAC=EEVIN	007200
	IF(HH-SEVIM) 8001,8022,8001	007210
8001	IF(PM) 8031,8033	007220
8033	IF(REM)8031,8032,8032	007230
8031	IF(MN)8022,8022,452	007240
8032	IF(REM-SEVIM) 8041,8042,8043	007250
8041	IF(PP-SEVIM) 8043,8043,8072	007260
8072	IF(SS) 8073,8074,8074	007270
8073	SS=EEVIN	007280
	GO TO 8022	007290
8074	IF(SS-SEVIM) 8042,8042,8075	007300
8075	SS=-SEVIM	007310
	PP=EEVIN	007320
	REM=-SEVIM	007330
	GO TO 8033	007340
8042	SS=SS-SEVIM	007350
	IF(SS)8051,8052,8053	007360
8051	SS=SEVIM	007370
	GO TO 8022	007380
8052	SS=2.0D0	007390
	GO TO 8022	007400
8053	SS=-SEVIM	007410
	REM=-SEVIM	007420
	GO TO 8033	007430



8043	PP=PP-SEVIM			007440
	IF(PP) 8061,8062,8063			007450
8061	PP=SEVIM			007460
	GO TO 8022			007470
8062	PP=2.000			007480
	GO TO 8022			007490
8063	PP=EEVIN			007500
	REM=-SEVIM			007510
	GO TO 8033			007520
452	PP=EEVIN			007530
	RR=EEVIN			007540
	SS=-SEVIM			007550
8022	L=1			007560
	M=1			007570
	AA=EEVIN			007580
	BB=EEVIN			007590
	CC=EEVIN			007600
	FF=EEVIN			007610
	GG=EEVIN			007620
	KK=0			007630
	KA=0			007640
	KKK=0			007650
	ADOM=BDOM			007660
	CR=BR			007670
	OMEGA(1)=OMEGAI			007680
	OMEGAO=OMEGAI			007690
	DOM=ADOM			007700
7040	IF(HH-SEVIM) 8021,7040,8021			007710
8191	IF(SS) 8191,8192,8192			007720
8192	IF(PP-SEVIM) 8091,8092,8093			007730
8091	IF(SS-SEVIM) 8291,8292,8293			007740
8081	WRITE(6,8081)			007750
	FORMAT(IH1,///18HMETHOD A	FAILED. ///		007760
	GO TO 8021			007770
8092	WRITE(6,8082)			007780
8082	FORMAT(IH1,///18HMETHOD B	FAILED. ///		007790
	GO TO 8021			007800
8093	WRITE(6,8084)			007810
8084	FORMAT(IH1,///18HMETHOD C	FAILED. ///		007820
	GO TO 8021			007830
8291	WRITE(6,8281)			007840
8281	FORMAT(IH1,///18HMETHOD D	FAILED. ///		007850
	GO TO 8021			007860
8292	WRITE(6,8282)			007870
8282	FORMAT(IH1,///18HMETHOD E	FAILED. ///		007880
	GO TO 8021			007890
8293	WRITE(6,8283)			007900
8283	FORMAT(IH1,///18HMETHOD F	FAILED. ///		007910





```

8021 IF(PM) 453,453,8035
8035 IF(REM) 8037,8036,8036
8036 IF(JK) 998,999,998
8037 REM=ERM
453 IF(PM) 6081,6082,6080
6081 IF(PMM-SEVIM) 6084,6080,6084
6084 IF(PMM*0.1D0-10.1D0) 6086,6085,6086
6085 PM=SEVIM
6086 GO TO 8036
6086 PM=EEVIN
6082 IF(PMM*0.1D0-1.1D0) 6080,6080,6085
6080 IF(MN) 9001,9001,8034
8034 MN=0
PM=PPM
GO TO 998
9001 JJ=JJ+1
DO 454 I=1,NS
454 P(I)=EEVIN
Z(I)=EEVIN
GO TO 450
600 STOP
END

```

```

007920
007930
007940
007950
007960
007970
007980
007990
008000
008010
008020
008030
008040
008050
008060
008070
008080
008090
008100
008110
008120
008130
008135

```

```

SUBROUTINE SUBSEC (RHO,THETA,D,TL,HM,Z,OMEGAI,HMH)
IMPLICIT REAL*8(A-H,O-Z)
RHOD=DABS(RHO)
IF(HMH) 41,40,41
40 IF(HM-5.0D0) 8,8,7
8 IF(OMEGAI-800.0D0) 10,7,7
7 HM=6.0D0
GO TO 10
41 HM=HMH
10 RA=TL/D
3 IF(RHOD-0.0D0) 3,3,4
5 IF(RA-1.0D0) 5,5,6
Z=0.0D0
RETURN
6 IF(RA-3.0D0) 15,15,16
15 Z=2.0D0
RETURN
16 IF(RA-6.0D0) 17,17,18
17 Z=3.0D0
RETURN
18 IF(HM-3.0D0) 1,1,2
1 Z=6.0D0
RETURN

```

```

008140
008150
008160
008170
008180
008190
008200
008210
008220
008230
008240
008250
008260
008270
008280
008290
008300
008310
008320
008330
008340
008350
008360

```



```

2 Z=HM*2.0D0
  RETURN
4 RD=RHOD/D
  IF(RD-1.0D0) 19,19,23
19 Z=0.0D0
  RETURN
23 IF(RD-3.0D0) 24,24,25
24 IF(THETA-0.44D0) 26,26,27
26 Z=0.0D0
  RETURN
27 IF(THETA-1.57D0) 28,28,29
28 Z=4.0D0
  RETURN
29 Z=6.0D0
  RETURN
25 IF(RD-6.0D0) 30,30,31
30 IF(THETA-0.44D0) 32,32,33
32 Z=4.0D0
  RETURN
33 IF(THETA-1.57D0) 34,34,35
34 Z=6.0D0
  RETURN
35 Z=8.0D0
  RETURN
31 IF(THETA-0.44D0) 36,36,3
36 Z=4.0D0
  RETURN
  END

```

```

008370
008380
008390
008400
008410
008420
008430
008440
008450
008460
008470
008480
008490
008500
008510
008520
008530
008540
008550
008560
008570
008580
008590
008600
008610
008620
008630
008640

```

```

SUBROUTINE DISTM (AMU,AIY,OMEGA,TL,R,D,DI,G,SD,RI,E,FFAC,U)

```

```

1 IMPLICIT REAL*8(A-H,O-Z)
2 DIMENSION U(6,6)
3 AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)
4 AREA=AREA*FFAC
5 BE=TL*OMEGA*DSQRT(AMU/(E*AREA))
6 P4=R*AMU*TL**4*OMEGA**2
7 IF(RI) 1,1,2
8 T=R*AMU*(AIY*OMEGA*TL)**2
9 GO TO 3
10 T=0.0D0
11 IF(SD) 4,4,5
12 S={AMU*(OMEGA*TL)**2}/(G*AREA)
13 GO TO 6
14 S=0.0D0
15 SPT=S+T
16 VV=DSQRT(P4+(S-T)**2/4.0D0)
17 AL1=DSQRT(VV-SPT*0.5D0)

```

```

008670
008680
008690
008700
008710
008720
008730
008740
008750
008760
008770
008780
008790
008800
008810
008820
008830
008840

```



```

AL2=DSQRT(VV+SPT*0.5D0)
CL=1.0D0/(AL1**2+AL2**2)
CHL1=(DEXP(AL1)+1.0D0/DEXP(AL1))/2.0D0
SHL1=(DEXP(AL1)-1.0D0/DEXP(AL1))/2.0D0
CL2=DCOS(AL2)
SL2=DSIN(AL2)
SBE=DSIN(BE)
CBE=DCOS(BE)
C00=CL*(AL2**2*CHL1+AL1**2*CL2)
C01=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)
C02=CL*(CHL1-CL2)
C03=CL*(SHL1/AL1-SL2/AL2)
DO 7 J=1,6
DO 7 K=1,6
U(J,K)=0.0D0
U(1,1)=CBE
U(1,6)=TL*SBE/(BE*ARA*E)
U(2,2)=C00-S*C02
U(2,3)=TL*(C01-SPT*C03)
U(2,4)=R*C02*TL**2
U(2,5)=(R*TL**3/P4)*((P4+S**2)*C03-S*C01)
U(3,2)=P4*C03/TL
U(3,3)=C00-T*C02
U(3,4)=TL*R*(C01-T*C03)
U(3,5)=U(2,4)
RR=1.0D0/R
U(4,2)=P4*RR*C02/TL**2
U(4,3)=RR*((P4+T**2)*C03-T*C01)/TL
U(4,4)=U(3,3)
U(4,5)=U(2,3)
U(5,2)=P4*RR*(C01-S*C03)/TL**3
U(5,3)=U(4,2)
U(5,4)=U(3,2)
U(5,5)=U(2,2)
U(6,1)=-AMU*TL*OMEGA**2*SBE/BE
U(6,6)=CBE
RETURN
END

```

```

SUBROUTINE DISTMO (AMU,AIX,AIY,OMEGA,TL,AJT,R,D,DI,G,SD,RI,FFAC,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
W=1.0D0/(G*AJT)
AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
BE=+DSQRT(AMU*(TL*AIX*OMEGA)**2*W)
P4=R*AMU*TL**4*OMEGA**2
IF(RI) 2,2,3

```



```

2 T=R*AMU*(AIY*OMEGA*TL)**2
3 GO TO 4
4 T=0.0D0
5 IF(SD) 5,5,6
6 S=(AMU*(OMEGA*TL)**2)/(G*AREA)
7 GO TO 7
8 S=0.0D0
9 VV=DSQRT(P4+(S-T)**2/4.0D0)
10 SPT=S+T
11 AL1=DSQRT(VV-SPT*0.5D0)
12 AL2=DSQRT(VV+SPT*0.5D0)
13 CL=1.0D0/(AL1**2+AL2**2)
14 CHL1=(DEXP(AL1)+1.0D0/DEXP(AL1))/2.0D0
15 SHL1=(DEXP(AL1)-1.0D0/DEXP(AL1))/2.0D0
16 CL2=DCOS(AL2)
17 SL2=DSIN(AL2)
18 SBE=DSIN(BE)
19 CBE=DCOS(BE)
20 CU0=CL*(AL2**2*CHL1+AL1**2*CL2)
21 CO1=CL*((AL2**2*SHL1)/AL1+(AL1**2*SL2)/AL2)
22 CO2=CL*(CHL1-CL2)
23 CO3=CL*(SHL1/AL1-SL2/AL2)
24 DO 1 J=1,6
25 DO 1 K=1,6
26 U(J,K)=0.0D0
27 U(1,1)=CBE
28 U(1,2)=TL*W*SBE/BE
29 U(2,1)=-AMU*TL*AIX**2*SBE/BE
30 U(2,2)=CBE
31 U(3,3)=CO0-S*CO2
32 U(3,4)=TL*(CO1-SPT*CO3)
33 U(3,5)=R*CO2*TL**2
34 U(3,6)=(R*TL**3/P4)*(-S*CO1+(P4+S**2)*CO3)
35 U(4,3)=P4*CO3/TL
36 U(4,4)=CO0-T*CO2
37 U(4,5)=TL*R*(CO1-T*CO3)
38 U(4,6)=U(3,5)
39 RR=1.0D0/R
40 U(5,3)=P4*RR*CO2/TL**2
41 U(5,4)=RR*(-T*CO1+(P4+T**2)*CO3)/TL
42 U(5,5)=U(4,4)
43 U(5,6)=U(3,4)
44 U(6,3)=P4*RR*(CO1-S*CO3)/TL**3
45 U(6,4)=U(5,3)
46 U(6,5)=U(4,3)
47 U(6,6)=U(3,3)
48 RETURN
49 END

```

```

009330
009340
009350
009360
009370
009380
009390
009400
009410
009420
009430
009440
009450
009460
009470
009480
009490
009500
009510
009520
009530
009540
009550
009560
009570
009580
009590
009600
009610
009620
009630
009633
009634
009635
009636
009637
009638
009639
009640
009641
009642
009643
009644
009645
009646
009647
009648
009649

```





```

SUBROUTINE RIGID (AMU,TL,OMEGA,AIY,RI,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
AM=AMU*TL
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
U(1,1)=1.0D0
U(6,1)=-AM*OMEGA**2
U(6,6)=1.0D0
U(2,2)=1.0D0
U(2,3)=TL
U(3,3)=1.0D0
U(4,2)=AM*TL*OMEGA**2/2.0D0
IF(RI) 2,2,3
2 U(4,3)=AM*OMEGA**2*(TL**2/6.0D0-AIY**2)
GO TO 4
3 U(4,3)=AM*(OMEGA*TL)**2/6.0D0
4 U(4,4)=1.0D0
U(4,5)=TL
U(5,2)=-U(6,1)
U(5,3)=U(4,2)
U(5,5)=1.0D0
RETURN
END

```

009652  
009653  
009654  
009655  
009656  
009657  
009658  
009659  
009660  
009661  
009662  
009663  
009664  
009665  
009666  
009667  
009668  
009669  
009670  
009671  
009672  
009673  
009674  
009675  
009676

```

SUBROUTINE RIGIO (AMU,TL,AIX,OMEGA,AIY,AJT,SDRI,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
DO 2 J=1,6
DO 2 K=1,6
2 U(J,K)=0.0D0
AM=AMU*TL
U(1,1)=1.0D0
U(2,1)=-AMU*TL*(AIX*OMEGA)**2
U(2,2)=1.0D0
U(3,3)=1.0D0
U(3,4)=TL
U(4,4)=1.0D0
U(5,3)=AM*TL*OMEGA**2/2.0D0
IF(SDRI-O.0D0) 30,30,31
30 U(5,4)=AM*OMEGA**2*(TL**2/6.0D0-AIY**2)
GO TO 1
31 U(5,4)=AM*OMEGA**2*(TL**2/6.0D0)
1 U(5,5)=1.0D0

```

009679  
009680  
009681  
009682  
009683  
009684  
009685  
009686  
009687  
009688  
009689  
009690  
009691  
009692  
009693  
009694  
009695  
009696  
009697



009698  
009699  
009700  
009710  
009720  
009730

U(5,6)=TL  
U(6,3)=AM\*OMEGA\*\*2  
U(6,4)=U(5,3)  
U(6,6)=1.0D0  
RETURN  
END

009760  
009762  
009764  
009770  
009780  
009790  
009800  
009810  
009820  
009830  
009840  
009850  
009860  
009870  
009880  
009890  
009900  
009910  
009920  
009930  
009940  
009950  
009960  
009970

SUBROUTINE STIFCO (THETA,RHO,U)  
IMPLICIT REAL\*8(A-H,O-Z)  
DIMENSION U(6,6)  
DO 1 J=1,6  
DO 1 K=1,6  
1 U(J,K)=0.0D0  
2 IF(RHO) 2,2,3  
2 V=-1.0D0  
3 GO TO 4  
3 V=1.0D0  
CT=DCOS(THETA)  
ST=DSIN(THETA)  
U(1,1)=CT  
U(1,2)=-V\*ST  
U(2,1)=V\*ST  
U(2,2)=CT  
U(3,3)=1.0D0  
U(4,4)=1.0D0  
U(5,5)=CT  
U(5,6)=-V\*ST  
U(6,5)=V\*ST  
U(6,6)=CT  
RETURN  
END

010000  
010010  
010020  
010030  
010040  
010050  
010060  
010070  
010080  
010090  
010100  
010110  
010120  
010130

SUBROUTINE STIFOO (THETA,RHO,U)  
IMPLICIT REAL\*8(A-H,O-Z)  
DIMENSION U(6,6)  
IF(RHO) 1,1,2  
1 V=-1.0D0  
2 GO TO 3  
2 V=1.0D0  
3 CT=DCOS(THETA)  
ST=DSIN(THETA)  
DO 4 J=1,6  
DO 4 K=1,6  
4 U(J,K)=0.0D0  
U(1,1)=CT  
U(1,4)=ST\*V



010140  
010150  
010160  
010170  
010180  
010190  
010200  
010210  
010220  
010230

U(2,2)=CT  
U(2,5)=ST\*V  
U(3,3)=1.000  
U(4,1)=-ST\*V  
U(4,4)=CT  
U(5,2)=-ST\*V  
U(5,5)=CT  
U(6,6)=1.000  
RETURN  
END

010260  
010270  
010280  
010290  
010300  
010310  
010320  
010330  
010340  
010350  
010360  
010370  
010380  
010390  
010400  
010410  
010420  
010430  
010440  
010450  
010460  
010470  
010480  
010490  
010500  
010510  
010520  
010530  
010540  
010550

SUBROUTINE STAVEC(SV,N,BC)  
IMPLICIT REAL\*8(A-H,O-Z)  
DIMENSION BC(6,22)  
DO 10 J=1,6  
10 BC(J,N)=0.000  
1 IF(SV-1.000) 1,1,2  
1 BC(4,N)=1.000  
1 BC(5,N)=1.000  
1 BC(6,N)=1.000  
RETURN  
2 IF(SV-2.000) 3,3,4  
2 BC(1,N)=1.000  
2 BC(2,N)=1.000  
2 BC(3,N)=1.000  
RETURN  
4 IF(SV-3.000) 5,5,6  
4 BC(3,N)=1.000  
4 BC(5,N)=1.000  
4 BC(6,N)=1.000  
RETURN  
6 IF(SV-4.000) 7,7,8  
6 BC(1,N)=1.000  
6 BC(3,N)=1.000  
6 BC(5,N)=1.000  
RETURN  
8 BC(2,N)=1.000  
8 BC(3,N)=1.000  
8 BC(6,N)=1.000  
RETURN  
END



```

SUBROUTINE STAVED (SV,N,BC)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION BC(6,22)
DO 10 L=1,6
  BC(L,N)=0.0D0
  IF(SV-1.0D0) 1,1,2
1  BC(2,N)=1.0D0
  BC(5,N)=1.0D0
  BC(6,N)=1.0D0
  RETURN
2  IF(SV-2.0D0) 3,3,4
  BC(1,N)=1.0D0
  BC(3,N)=1.0D0
  BC(4,N)=1.0D0
  RETURN
4  IF(SV-3.0D0) 5,5,6
  BC(2,N)=1.0D0
  BC(4,N)=1.0D0
  BC(6,N)=1.0D0
  RETURN
6  IF(SV-4.0D0) 7,7,5
  BC(1,N)=1.0D0
  BC(4,N)=1.0D0
  BC(6,N)=1.0D0
  RETURN
END

```

010580  
010590  
010600  
010610  
010620  
010630  
010640  
010650  
010660  
010670  
010680  
010690  
010700  
010710  
010720  
010730  
010740  
010750  
010760  
010770  
010780  
010790  
010800  
010810  
010820  
010830

```

SUBROUTINE INVERT (A,N,D,L,M)
IMPLICIT REAL*8(A-H,O-Z)
PROGRAM FOR FINDING THE INVERSE OF A NXN MATRIX
DIMENSION A(25,25),L(25),M(25)
SEARCH FOR LARGEST ELEMENT
D=1.0D0
DO 80 K=1,N
  L(K)=K
  M(K)=K
  BIGA=A(K,K)
DO 20 I=K,N
DO 20 J=K,N
  IF(DABS(BIGA)-DABS(A(I,J))) 10,20,20
10 BIGA=A(I,J)
  L(K)=I
  M(K)=J
20 CONTINUE
INTERCHANGE ROWS
J=L(K)
IF(L(K)-K) 35,35,25

```

010860  
010870  
010880  
010890  
010900  
010910  
010920  
010930  
010940  
010950  
010960  
010970  
010980  
010990  
011000  
011010  
011020  
011030  
011040  
011050





```

25 DO 30 I=1,N
   HOLD=-A(K,I)
   A(K,I)=A(J,I)
30 A(J,I)=HOLD
   INTERCHANGE COLUMNS
35 I=M(K)
   IF(M(K)-K) 45,45,37
37 DO 40 J=1,N
   HOLD=-A(J,K)
   A(J,K)=A(J,I)
40 A(J,I)=HOLD
   DIVIDE COLUMN BY MINUS PIVOT
45 DO 55 I=1,N
   IF(I-K) 50,55,50
46 A(I,K)=A(I,K)/(-A(K,K))
50 CONTINUE
55 REDUCE MATRIX
   DO 65 I=1,N
   DO 65 J=1,N
56 IF(I-K) 57,65,57
57 IF(J-K) 60,65,60
60 A(I,J)=A(I,K)*A(K,J)+A(I,J)
65 CONTINUE
   DIVIDE ROW BY PIVOT
   DO 75 J=1,N
68 IF(J-K) 70,75,70
70 A(K,J)=A(K,J)/A(K,K)
75 CONTINUE
   CONTINUED PRODUCT OF PIVOTS
   D=D*A(K,K)
   REPLACE PIVOT BY RECIPROCAL
   A(K,K)=1.0D0/A(K,K)
80 CONTINUE
   FINAL ROW AND COLUMN INTERCHANGE
   K=N
100 K=K-1
   IF(K) 150,150,103
103 I=L(K)
   IF(I-K) 120,120,105
105 DO 110 J=1,N
   HOLD=A(J,K)
   A(J,K)=-A(J,I)
110 A(J,I)=HOLD
120 J=M(K)
   IF(J-K) 100,100,125
125 DO 130 I=1,N
   HOLD=A(K,I)
   A(K,I)=-A(J,I)

```

```

011060
011070
011080
011090
011100
011110
011120
011130
011140
011150
011160
011170
011180
011190
011200
011210
011220
011230
011240
011250
011260
011270
011280
011290
011300
011310
011320
011330
011340
011350
011360
011370
011380
011390
011400
011410
011420
011430
011440
011450
011460
011470
011480
011490
011500
011510
011520
011530

```



```

130 A(J,I)=HOLD
GO TO 100
150 RETURN
END

```

011540  
011550  
011560  
011570

```

SUBROUTINE HANGER (CLX,CLY,CTZ,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
ZZ=1.0D0
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
U(1,1)=ZZ
U(2,2)=ZZ
U(3,3)=ZZ
U(4,4)=ZZ
U(5,5)=ZZ
U(6,6)=ZZ
U(4,3)=CTZ
U(5,2)=-CLY
U(6,1)=CLX
RETURN
END

```

011600  
011610  
011620  
011630  
011640  
011650  
011660  
011670  
011680  
011690  
011700  
011710  
011720  
011730  
011740  
011750  
011760  
011770

```

SUBROUTINE HANGED (CTX,CLZ,CTY,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6)
ZZ=1.0D0
DO 1 J=1,6
DO 1 K=1,6
1 U(J,K)=0.0D0
U(1,1)=ZZ
U(2,1)=CTX
U(2,2)=ZZ
U(3,3)=ZZ
U(4,4)=ZZ
U(5,4)=CTY
U(5,5)=ZZ
U(6,3)=-CLZ
U(6,6)=ZZ
RETURN
END

```

011800  
011810  
011820  
011830  
011840  
011850  
011860  
011870  
011880  
011890  
011900  
011910  
011920  
011930  
011940  
011950  
011960  
011970



```

SUBROUTINE CFIELO(AJT,R,Z,G,SDRI,RHO,THETA,D,DI,FFAC,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
PHI=THETA/Z
RHOD=RHO
IF(RHO-ZA) 11,11,12
11 V=-ZZ
GO TO 1
12 V=ZZ
1 CP=DCOS(PHI)
SP=DSIN(PHI)
W=ZZ/(G*AJT)
F1=(PHI*CP+SP)/2.0D0
F3=(SP-CP*PHI)/2.0D0
F5=(2.0D0-2.0D0*CP-PHI*SP)/2.0D0
F6=(2.0D0*PHI+PHI*CP-3.0D0*SP)/2.0D0
RHO=DABS(RHO)
DO 2 J=1,6
DO 2 K=1,6
2 A(J,K)=ZA
13 IF(SDRI) 13,13,14
13 AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
SD=RHO*THETA/(G*Z*AREA)
GO TO 15
14 SD=ZA
15 A(1,1)=CP
A(1,2)=(W*RHO*F1)-(RHO*F3*R)
A(1,4)=V*SP
A(1,5)=V*(W+R)*(RHO*PHI*SP)/2.0D0
A(1,6)=V*(W+R)*(RHO**2)*F3
A(2,2)=CP
A(2,5)=V*SP
A(2,6)=V*RHO*(ZZ-CP)
A(3,1)=-A(2,6)
A(3,2)=-ZZ*A(1,6)
A(3,3)=ZZ
A(3,4)=RHO*SP
A(3,5)=(R*(RHO**2)*PHI*SP)/2.0D0-W*RHO**2*F5
A(3,6)=(R*(RHO**3*F3)-(W*F6*RHO**3)-SD
A(4,1)=-V*SP
A(4,2)=-V*(W+R)*RHO*PHI*SP/2.0D0
A(4,4)=CP
A(4,5)=(R*RHO*F1)-(W*RHO*F3)
A(4,6)=A(3,5)
A(5,2)=-V*SP
A(5,5)=CP

```

012000  
012010  
012020  
012030  
012040  
012050  
012060  
012070  
012080  
012090  
012100  
012110  
012120  
012130  
012140  
012150  
012160  
012170  
012180  
012190  
012200  
012210  
012220  
012230  
012240  
012250  
012260  
012270  
012280  
012290  
012300  
012310  
012320  
012330  
012340  
012350  
012360  
012370  
012380  
012390  
012400  
012410  
012420  
012430  
012440  
012450  
012460  
012470



012480  
012490  
012500  
012510  
012520

A(5,6)=A(3,4)  
A(6,6)=ZZ  
RHO=RHOD  
RETURN  
END

012550  
012560  
012570  
012580  
012590  
012600  
012610  
012620  
012630  
012640  
012650  
012660  
012670  
012680  
012690  
012700  
012710  
012720  
012730  
012740  
012750  
012760  
012770

```

SUBROUTINE POINO(AMU,AIX,AIY,OMEGA,TL,Z,SDRI,B)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(6,6)
ZZ=1.0D0
ZA=0.0D0
AM=AMU*TL/(Z-ZZ)
DO 1 J=1,6
DO 1 K=1,6
1 B(J,K)=ZA
B(1,1)=ZZ
B(2,1)=-AM*(AIX*OMEGA)**2
B(2,2)=ZZ
B(3,3)=ZZ
B(4,4)=ZZ
IF(SDRI-ZA) 9,9,10
9 B(5,4)=-AM*(AIY*OMEGA)**2
GO TO 11
10 B(5,4)=ZA
11 B(5,5)=ZZ
B(6,3)=AM*OMEGA**2
B(6,6)=ZZ
RETURN
END

```

012800  
012810  
012820  
012830  
012840  
012850  
012860  
012870  
012880  
012890  
012900  
012910  
012920  
012930  
012940  
012950

```

SUBROUTINE CFIELD(R,Z,G,SD,RHO,THETA,D,DI,E,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
PHI=THETA/Z
RD=DABS(RHO)
IF(RHO)1,1,2
1 V=-ZZ
GO TO 3
2 V=ZZ
3 CP=DCOS(PHI)
SP=DSIN(PHI)
DO 4 J=1,6
DO 4 K=1,6
4 A(J,K)=ZA

```





012960  
012970  
012980  
012990  
013000  
013010  
013020  
013030  
013040  
013050  
013060  
013070  
013080  
013090  
013100  
013110  
013120  
013130  
013140  
013150  
013160  
013170  
013180  
013190  
013200  
013210  
013220  
013230  
013240  
013250  
013260

```

A(1,1)=CP
A(1,2)=-V*SP
A(1,3)=-V*RD*(ZZ-CP)
A(1,4)=-V*RD**2*R*(PHI-SP)
AR=3.141592654D0*(D**2-DI**2)/4.0D0
W=ZZ/(E*AR)
F3=(SP-PHI*CP)*0.50D0
F1=(PHI*CP+SP)*0.50D0
F5=(2.0D0-2.0D0*CP-PHI*SP)*0.50D0
F6=(2.0D0*PHI+PHI*CP-3.0D0*SP)*0.50D0
A(1,5)=V*(RD*W*PHI*SP*0.50D0-R*F5*RD**3)
A(1,6)=RD*W*F1+R*F6*RD**3
A(2,1)=V*SP
A(2,2)=CP
A(2,3)=RD*SP
A(2,4)=R*RD**2*(ZZ-CP)
A(2,5)=RD*F3*(W+R*RD**2)
A(2,6)=A(1,5)
A(3,3)=ZZ
A(3,4)=RD*R*PHI
A(3,5)=A(2,4)
A(3,6)=A(1,4)
A(4,4)=ZZ
A(4,5)=A(2,3)
A(4,6)=A(1,3)
A(5,5)=CP
A(5,6)=A(1,2)
A(6,5)=A(2,1)
A(6,6)=CP
RETURN
END

```

SUBROUTINE POINT(AMU,AIY,OMEGA,TL,Z,RI,B)

```

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION B(6,6)
ZZ=1.0D0
ZA=0.0D0
AM=AMU*TL/(Z-ZZ)
DO 1 J=1,6
DO 1 K=1,6
1 B(J,K)=ZA
B(1,1)=ZZ
B(2,2)=ZZ
B(3,3)=ZZ
IF(RI)2,2,3
2 B(4,3)=-AM*(AIY*OMEGA)**2
3 B(4,4)=ZZ
013290
013300
013310
013320
013330
013340
013350
013360
013370
013380
013390
013400
013410
013420
013430

```



013440  
013450  
013460  
013470  
013480  
013490

B(5,2)=AM\*OMEGA\*\*2  
B(5,5)=ZZ  
B(6,1)=-B(5,2)  
B(6,6)=ZZ  
RETURN  
END

013520  
013530  
013540  
013550  
013560  
013570  
013580  
013590  
013600  
013610  
013620  
013630  
013640  
013650  
013660  
013670  
013680  
013690  
013700  
013710  
013720  
013725  
013740  
013750  
013760  
013770  
013780  
013790  
013800  
013810  
013820  
013830  
013840  
013850  
013860  
013870  
013880

SUBROUTINE SFIELD(DI,TL,R,Z,G,SD,D,E,FFAC,A)  
IMPLICIT REAL\*8(A-H,O-Z)  
DIMENSION A(6,6)  
ZZ=1.0D0  
ZA=0.0D0  
AREA=(3.141592654D0/4.0D0)\*(D\*\*2-DI\*\*2)  
DL=TL/Z  
DO 1 J=2,5  
1 A(1,J)=ZA  
A(1,1)=ZZ  
A(1,6)=DL/(E\*ARA)  
A(2,1)=ZA  
A(2,2)=ZZ  
A(2,3)=DL\*\*2\*R/2.0D0  
A(2,4)=DL\*\*2\*R/2.0D0  
IF(SD) 2,2,3  
2 A(2,5)=DL\*\*3\*R/6.0D0-DL/(G\*AREA)  
GO TO 4  
3 A(2,5)=DL\*\*3\*R/6.0D0  
4 A(2,6)=ZA  
A(3,1)=ZA  
A(3,2)=ZA  
A(3,3)=ZZ  
A(3,4)=DL\*R  
A(3,5)=A(2,4)  
A(3,6)=ZA  
DO 5 J=1,6  
5 A(4,J)=ZA  
A(5,J)=ZA  
A(6,J)=ZA  
A(4,4)=ZZ  
A(4,5)=DL  
A(5,5)=ZZ  
A(6,6)=ZZ  
RETURN  
END



```

SUBROUTINE SFIELO(DI,TL,AJT,R,Z,G,SDRI,D,FFAC,A)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6)
ZZ=1.0D0
ZA=0.0D0
AREA=(3.141592654D0/4.0D0)*(D**2-DI**2)*FFAC
DL=TL/Z
DO 2 J=1,6
DO 2 K=1,6
2 A(J,K)=ZA
A(1,1)=ZZ
A(2,2)=ZZ
A(3,3)=ZZ
A(4,4)=ZZ
A(5,5)=ZZ
A(6,6)=ZZ
IF(SDRI-ZA) 7,7,8
7 A(3,6)=DL**3*R/6.0D0-DL/(G*AREA)
GO TO 1
8 A(3,6)=DL**3*R/6.0D0
1 A(1,2)=DL/(G*AJT)
A(3,4)=DL
A(3,5)=DL**2*R/2.0D0
A(4,5)=DL*R
A(4,6)=A(3,5)
A(5,6)=DL
RETURN
END

```

013910  
013920  
013930  
013940  
013950  
013960  
013970  
013980  
013990  
014000  
014010  
014020  
014030  
014040  
014056  
014060  
014070  
014080  
014090  
014100  
014110  
014120  
014130  
014140  
014150  
014160  
014170  
014180

```

SUBROUTINE MATMUL(A,B,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(6,6),B(6,6),C(6,6),U(6,6)
DO 1 J=1,6
DO 1 K=1,6
C(J,K)=0.0D0
DO 1 L=1,6
1 C(J,K)=C(J,K)+B(J,L)*A(L,K)
DO 2 J=1,6
DO 2 K=1,6
2 U(J,K)=C(J,K)
RETURN
END

```

014210  
014220  
014230  
014240  
014250  
014260  
014270  
014280  
014290  
014300  
014310  
014320  
014330



```

SUBROUTINE FINBRA(U,VV,V,MAT,A,Z)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6),A(6,6),VV(6,3),V(6,3)
IF(MAT) 65,64,65
64 DO 67 J=1,6
DO 67 K=1,3
V(J,K)=0.0D0
DO 67 L=1,6
67 V(J,K)=V(J,K)+U(J,L)*VV(L,K)
RETURN
65 N=Z-1.0D0
DO 5 M=1,N
DO 4 J=1,6
DO 4 K=1,3
V(J,K)=0.0D0
DO 4 L=1,6
4 V(J,K)=V(J,K)+U(J,L)*VV(L,K)
DO 5 J=1,6
DO 5 K=1,3
5 VV(J,K)=V(J,K)
DO 7 J=1,6
DO 7 K=1,3
V(J,K)=0.0D0
DO 7 L=1,6
7 V(J,K)=V(J,K)+A(J,L)*VV(L,K)
RETURN
END

```

014360  
014370  
014380  
014390  
014400  
014410  
014420  
014430  
014440  
014450  
014460  
014470  
014480  
014490  
014500  
014510  
014520  
014530  
014540  
014550  
014560  
014570  
014580  
014590  
014600  
014610  
014620

```

SUBROUTINE FINMAT(U,UU,V,MAT,A,Z)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION U(6,6),A(6,6),V(6,3),UU(6,3)
ZA=0.0D0
IF(MAT) 65,64,65
64 DO 67 J=1,6
DO 67 K=1,3
V(J,K)=ZA
DO 67 L=1,6
67 V(J,K)=V(J,K)+U(J,L)*UU(L,K)
RETURN
65 N=Z-1.0D0
DO 5 M=1,N
DO 4 J=1,6
DO 4 K=1,3
V(J,K)=ZA
DO 4 L=1,6
4 V(J,K)=V(J,K)+U(J,L)*UU(L,K)
DO 5 J=1,6

```

014650  
014660  
014670  
014680  
014690  
014700  
014710  
014720  
014730  
014740  
014750  
014760  
014770  
014780  
014790  
014800  
014810  
014820  
014830





014840  
014850  
014860  
014870  
014880  
014890  
014900  
014910  
014920

```

5 DO 5 K=1,3
  UU(J,K)=V(J,K)
  DC 7 J=1,6
  DO 7 K=1,3
    V(J,K)=ZA
  DO 7 L=1,6
    V(J,K)=V(J,K)+A(J,L)*UU(L,K)
  7 RETURN
  END

```

014950  
014960  
014970  
014980  
014990  
015000  
015010  
015020  
015030  
015040  
015050  
015060  
015070  
015080  
015090  
015100  
015110  
015120  
015130  
015140  
015150  
015160  
015170  
015180  
015190  
015200  
015210  
015220  
015230  
015240  
015250  
015260  
015270  
015280  
015290  
015300  
015310

```

SUBROUTINE BRANCH(R3,VV,PHI,U)
  IMPLICIT REAL*8(A-H,O-Z)
  DIMENSION R3(3,3),VV(6,3),U(6,6),R1(25,25),LL(25),MM(25),R2(3,3),
  1 R(3,3),G1(3,3),G2(3,3)
  ZZ=1.0D0
  ZA=0.0D0
  DO 4 L=1,3
    R1(1,L)=VV(1,L)
    R1(2,L)=VV(2,L)
    R1(3,L)=VV(3,L)
    R2(1,L)=VV(4,L)
    R2(2,L)=VV(5,L)
    R2(3,L)=VV(6,L)
  4 SP=DSIN(PHI)
  CP=DCOS(PHI)
  G1(1,1)=CP
  G1(1,2)=-SP
  G1(1,3)=ZA
  G1(2,1)=SP
  G1(2,2)=CP
  G1(2,3)=ZA
  G1(3,1)=ZA
  G1(3,2)=ZA
  G1(3,3)=ZZ
  G2(1,1)=ZZ
  G2(1,2)=ZA
  G2(1,3)=ZA
  G2(2,1)=ZA
  G2(2,2)=CP
  G2(2,3)=-SP
  G2(3,1)=ZA
  G2(3,2)=SP
  G2(3,3)=CP
  CALL INVERT (R1,3,D,LL,MM)
  DO 5 J=1,3
    DO 5 K=1,3
      R3(J,K)=ZA

```



```

5 DO 5 L=1,3
  R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)
DO 6 J=1,3
  DO 6 K=1,3
    R1(J,K)=ZA
DO 6 L=1,3
  R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)
DO 7 J=1,3
  DO 7 K=1,3
    R(J,K)=ZA
DO 7 L=1,3
  R(J,K)=R(J,K)+R1(J,L)*R3(L,K)
7 DO 8 J=1,6
  DO 8 K=1,6
    U(J,K)=ZA
8 DO 1,1)=ZZ
  U(2,2)=ZZ
  U(3,3)=ZZ
  U(4,4)=ZZ
  U(5,5)=ZZ
  U(6,6)=ZZ
DO 9 L=1,3
  U(4,L)=R(1,L)
  U(5,L)=R(2,L)
  U(6,L)=R(3,L)
9 RETURN
END

```

015320  
015330  
015340  
015350  
015360  
015370  
015380  
015390  
015400  
015410  
015420  
015430  
015440  
015450  
015460  
015470  
015480  
015490  
015500  
015510  
015520  
015530  
015540  
015550  
015560  
015570  
015580

```

SUBROUTINE BRANCO(R3,VV,PHI,U)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R3(3,3),U(6,6),VV(6,3),G1(3,3),G2(3,3)
1,S(3,3),LL(25),MM(25)
ZZ=1.0D0
ZA=0.0D0
DO 4 K=1,3
  R1(1,K)=VV(1,K)
  R1(2,K)=VV(3,K)
  R1(3,K)=VV(4,K)
  R2(1,K)=VV(2,K)
  R2(2,K)=VV(5,K)
  R2(3,K)=VV(6,K)
4 SP=DSIN(PHI)
CP=DCOS(PHI)
G1(1,1)=CP
G1(1,2)=ZA
G1(1,3)=-SP
G1(2,1)=ZA

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```

G1(2,2)=ZZ
G1(2,3)=ZA
G1(3,1)=SP
G1(3,2)=ZA
G1(3,3)=CP
G2(1,1)=CP
G2(1,2)=SP
G2(1,3)=ZA
G2(2,1)=-SP
G2(2,2)=CP
G2(2,3)=ZA
G2(3,1)=ZA
G2(3,2)=ZA
G2(3,3)=ZZ
CALL INVERT (R1,3,D,LL,MM)
DO 5 J=1,3
DO 5 K=1,3
R3(J,K)=ZA
DO 5 L=1,3
R3(J,K)=R3(J,K)+R1(J,L)*G1(L,K)
5 DO 6 J=1,3
DO 6 K=1,3
R1(J,K)=ZA
DO 6 L=1,3
R1(J,K)=R1(J,K)+G2(J,L)*R2(L,K)
6 DO 7 J=1,3
DO 7 K=1,3
S(J,K)=ZA
DO 7 L=1,3
S(J,K)=S(J,K)+R1(J,L)*R3(L,K)
7 DO 8 J=1,6
DO 8 K=1,6
U(J,K)=ZA
U(1,1)=ZZ
U(2,1)=S(1,1)
U(2,2)=ZZ
U(2,3)=S(1,2)
U(2,4)=S(1,3)
U(3,3)=ZZ
U(4,4)=ZZ
U(5,1)=S(2,1)
U(5,3)=S(2,2)
U(5,4)=S(2,3)
U(5,5)=ZZ
U(6,1)=S(3,1)
U(6,3)=S(3,2)
U(6,4)=S(3,3)
U(6,6)=ZZ

```



RETURN  
END

```

SUBROUTINE STATEM(SVI,UU,D1,D2,P1,P2,PP,PM,SS)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION SVI(6,1),UU(6,3)
ZZ=1.0D0
ZA=0.0D0
M=0
DO 1 J=1,6
DO 1 K=1,3
1 UU(J,K)=ZA
IF(PM) 40,40,41
40 DO 42 J=1,6
IF(SVI(J,1)) 42,42,43
43 M=M+1
UU(J,M)=ZZ
IF(M-1) 42,44,42
44 UU(J,2)=-D1
UU(J,3)=-D2
42 CONTINUE
RETURN
41 P1=P1+D1
P2=P2+D2
IF(SS)14,15,15
15 IF(SS-ZZ) 12,11,10
14 IF(PP-ZZ) 12,11,10
10 DO 4 J=1,6
IF(SVI(J,1)) 4,4,2
2 M=M+1
UU(J,M)=ZZ
IF(M-2) 4,3,5
3 UU(J,1)=P1
GO TO 4
5 UU(J,1)=P2
4 CONTINUE
RETURN
11 DO 21 J=1,6
IF(SVI(J,1))21,21,22
22 M=M+1
UU(J,M)=ZZ
IF(M-1) 25,26,25
26 UU(J,3)=P1
25 IF(M-2) 21,23,21
23 UU(J,3)=P2
21 CONTINUE
RETURN

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```
12 DO 31 J=1,6
   IF(SVI(J,1)) 31,31,32
32 M=M+1
   UU(J,M)=ZZ
   IF(M-1) 35,36,35
36 UU(J,2)=P2
35 IF(M-2) 31,31,33
33 UU(J,2)=P1
31 CONTINUE
   RETURN
   END
```

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```

SUBROUTINE STATEB(SVB,VV,N,YY,L,BRC,D1,D2,PP,PM,SS,Q1,Q2,Q3,FINK)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION VV(6,3),YY(3,3,25),V(3,3),SVB(6,22)
ZZ=1.0D0
ZA=0.0D0
B1=ZA
B2=ZA
B3=ZA
D1=ZA
D2=ZA
IF(PM) 33,40,22
IF(BRC) 21,20,21
22 IF(L-1) 23,20,23
21 IF(L-1) 23,20,23
23 DO 10 J=1,3
   DO 10 K=1,3
     V(J,K)=YY(J,K,N)
     IF(SS) 11,12,12
     IF(PP-ZZ) 32,31,30
     IF(SS-ZZ) 33,31,30
30 B1=V(1,1)+V(1,2)*D1+V(1,3)*D2+B1
   B2=V(2,1)+V(2,2)*D1+V(2,3)*D2+B2
   B3=V(3,1)+V(3,2)*D1+V(3,3)*D2+B3
   GO TO 33
31 B1=B1+V(1,1)*D2+V(1,3)*D1+V(1,2)
   B2=B2+V(2,1)*D2+V(2,3)*D1+V(2,2)
   B3=B3+V(3,1)*D2+V(3,3)*D1+V(3,2)
   GO TO 33
32 B1=B1+V(1,1)*D1+V(1,2)*D2+V(1,3)
   B2=B2+V(2,1)*D1+V(2,2)*D2+V(2,3)
   B3=B3+V(3,1)*D1+V(3,2)*D2+V(3,3)
33 Q1=Q1*B1
   Q2=Q1*Q2+B2
   Q3=Q1*Q3+B3
20 M=1
   DO 1 J=1,6
```



```

DO 1 K=1,3
1 VV(J,K)=ZA
DO 4 J=1,6
2 IF(SVB(J,N)) 4,4,2
3 IF(M-1)7,3,7
35 IF(PM) 34,35,35
35 VV(J,1)=Q1
GO TO 36
34 VV(J,1)=ZZ
36 M=M+1
GO TO 4
7 IF(M-2) 4,8,9
8 VV(J,1)=Q2
VV(J,M)=ZZ
M=M+1
GO TO 4
9 VV(J,1)=Q3
VV(J,M)=ZZ
4 CONTINUE
RETURN
40 IF(FINK) 54,51,54
54 IF(BRC) 51,51,52
52 DO 53 J=1,3
DO 53 K=1,3
53 V(J,K)=YY(J,K,N)
D1=(V(1,2)/V(1,1))+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.0D0
D2=(V(1,3)/V(1,1))+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.0D0
51 M=0
DO 41 J=1,6
DO 41 K=1,3
41 VV(J,K)=ZA
DO 42 J=1,6
IF(SVB(J,N)) 42,42,43
43 M=M+1
VV(J,M)=ZZ
IF(M-1) 42,44,42
44 VV(J,2)=-D1
VV(J,3)=-D2
42 CONTINUE
RETURN
END

```

```

SUBROUTINE BRCOR(R3,UU,XX,JK,VV,PM)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION R3(3,3),UU(6,3),XX(3,3),UUU(3,3),VV(6,3)
ZZ=1.0D0
ZA=0.0D0
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017670

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```

DO 92 K=1,3
92 V(L,K)=UU(N,K)
91 CONTINUE
64 IF(PM) 66,64,61
62 D1=(V(1,2)/V(1,1))+V(2,2)/V(2,1)+V(3,2)/V(3,1))/3.0D0
D2=(V(1,3)/V(1,1))+V(2,3)/V(2,1)+V(3,3)/V(3,1))/3.0D0
FINK=ZZ
RETURN
63 FINK=ZA
RETURN
66 RETURN
61 IF(SS) 311,312,312
311 IF(PP-ZZ) 400,401,402
312 IF(SS-ZZ) 313,314,315
402 IF(RR-ZZ) 300,403,300
315 IF(RR-ZZ) 340,341,340
300 IF(V(3,3)) 93,94,93
340 IF(V(3,2)) 93,94,93
93 DO 95 J=2,3
DO 95 K=1,3
95 D(J,K)=V(J,K)
DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)
IF(DV) 98,94,98
94 RR=ZZ
HH=ZZ
RETURN
403 IF(V(2,3)) 96,200,96
341 IF(V(2,2)) 96,200,96
96 DO 97 K=1,3
D(2,K)=V(3,K)
97 D(3,K)=V(2,K)
DV=V(1,2)*D(2,3)-V(1,3)*D(2,2)
IF(DV) 98,200,98
98 X1=X1-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV
X2=X2-(V(1,2)*D(2,1)-V(1,1)*D(2,2))/DV
IF(X1) 100,102,100
102 IF(X2) 100,101,100
101 FF=ZZ
RETURN
100 IF(SS) 354,355,355
354 Y2=Y2-D(3,1)/D(3,3)-X1*D(3,2)/D(3,3)
RETURN
355 Y1=Y1-D(3,1)/D(3,2)-X2*D(3,3)/D(3,2)
RETURN
99 IF(SS) 20,21,21
20 PP=ZZ
GO TO 23

```





21	SS=ZZ				018640
23	RR=ZA				018650
	HH=ZZ				018660
	RETURN				018670
314	IF(RR-ZZ)	330,331,330			018680
401	IF(RR-ZZ)	301,404,301			018690
330	IF(V(3,1))	81,82,81			018700
301	IF(V(3,2))	81,82,81			018710
81	DO 83	J=2,3			018720
	DO 83	K=1,3			018730
83	D(J,K)=V(J,K)				018740
	DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)				018750
	IF(DV)	86,82,86			018760
82	RR=ZZ				018770
	HH=ZZ				018780
	RETURN				018790
331	IF(V(2,1))	84,70,84			018800
404	IF(V(2,2))	84,70,84			018810
84	DO 85	K=1,3			018820
	D(2,K)=V(3,K)				018830
85	D(3,K)=V(2,K)				018840
	DV=V(1,1)*D(2,2)-V(1,2)*D(2,1)				018850
	IF(DV)	86,70,86			018860
86	X1=X1-(V(1,3)*D(2,2)-V(1,2)*D(2,3))/DV				018870
	X2=X2-(V(1,1)*D(2,3)-V(1,3)*D(2,1))/DV				018880
	IF(X1)	203,201,203			018890
201	IF(X2)	203,202,203			018900
202	FF=ZZ				018910
	RETURN				018920
203	IF(SS)	352,353,353			018930
353	Y1=Y1-D(3,3)/D(3,1)-X2*D(3,2)/D(3,1)				018940
	RETURN				018950
352	Y2=Y2-D(3,3)/D(3,2)-X1*D(3,1)/D(3,2)				018960
	RETURN				018970
70	IF(SS)	25,24,24			018980
24	SS=2.0D0				018990
	GO TO 26				019000
25	PP=2.0D0				019010
26	RR=ZA				019020
	HH=ZZ				019030
	RETURN				019040
313	IF(RR-ZZ)	320,321,320			019050
400	IF(RR-ZZ)	302,405,302			019060
320	IF(V(3,3))	71,72,71			019070
302	IF(V(3,1))	71,72,71			019080
71	DO 73	J=2,3			019090
	DO 73	K=1,3			019100
73	D(J,K)=V(J,K)				019110



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```

DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)
IF(DV) 76,72,76
72 RR=ZZ
   HH=ZZ
   RETURN
405 IF(V(2,1)) 74,99,74
321 IF(V(2,3)) 74,99,74
200 KKK=1
   RR=ZA
   RETURN
74 DO 75 K=1,3
   D(2,K)=V(3,K)
75 D(3,K)=V(2,K)
   DV=V(1,1)*D(2,3)-V(1,3)*D(2,1)
   IF(DV) 76,99,76
76 X2=X2-(V(1,2)*D(2,3)-V(1,3)*D(2,2))/DV
   X1=X1-(V(1,1)*D(2,2)-D(2,1)*V(1,2))/DV
211 IF(X1) 213,211,213
212 IF(X2) 213,212,213
   FF=ZZ
   RETURN
213 IF(SS) 350,351,351
351 Y1=Y1-(D(3,2)+X2*D(3,1))/D(3,3)
   RETURN
350 Y2=Y2-(D(3,2)+X1*D(3,3))/D(3,1)
   RETURN
END

```



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1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE  VIBRATION OF AN ELL-SHAPED UNIFORM FILAMENT WITH CLAMPED ENDS			
4. DESCRIPTIVE NOTES (Type of report and, inclusive dates) Master's Thesis; December 1970			
5. AUTHOR(S) (First name, middle initial, last name)  Cetin Derin, Lieutenant (junior grade), Turkish Navy			
6. REPORT DATE December 1970		7a. TOTAL NO. OF PAGES 116	7b. NO. OF REFS 3
8a. CONTRACT OR GRANT NO.		9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.			
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. DISTRIBUTION STATEMENT  This document has been approved for public release and sale; its distribution is unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY  Naval Postgraduate School Monterey, California 93940	
13. ABSTRACT  An analysis is made of in-plane and out-of-plane vibrations of a uniform filament which is bent at right angles and the ends of which are fixed. Shear deflection and rotatory inertia are neglected. Curve sheets are presented from which one may find the first several frequencies of vibration in terms of the geometrical and physical variables. An important application is to the vibrations of piping systems.  Also, as a separate project, a vibration program written by G. Fink for the CDC 1604 computer and modified by Y. S. Kim, has been rewritten for use with the IBM 360/67 computer.			



KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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PIPE VIBRATION						
VIBRATION OF ELL-SHAPED BAR						



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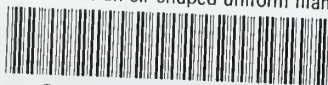
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